MAA
AMC AMERICAN MATHEMATICS COMPETITIONS

MAA American Mathematics Competitions
27th Annual

AMC 10 B

Friday, November 14, 2025

The problems and solutions for this AMC 10 B were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

INSTRUCTIONS | 说明

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
   在未收到监考老师开考指示前，请不要翻开此封面。
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
   这是一套包括25道选择题的测试。每道题目只有一个正确答案。
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet and appear sufficiently dark will be scored.
   请将每道题目的答案用＃2铅笔标注在答题卡上。请注意检查涂写的黑色圆圈的准确性，用橡皮完全擦掉错误的答案。只有在答题卡上规范填涂且颜色足够深的答案才会被计分。
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
   评分：每道题目答对得6分， 不答得1.5分，答错得0分。
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, computing devices, protractors, or graph paper are allowed. No problems in the competition will require the use of a calculator.
   只能使用空白的草稿纸、尺子、圆规和橡皮作为辅助工具。不允许的器具包括计算器、智能手表、手机、计算设备、量角器或坐标纸。竞赛中没有任何问题必须要使用计算器。
6. Figures are not necessarily drawn to scale.
   图形不一定按比例绘制。
7. Please bubble in carefully the mobile phone number used for registration. This mobile phone number will serve as your unique identification number. Please double-check it for accuracy, as the organizing committee will not accept appeals for scores invalidated due to a wrong number.
   请在答题卡上填涂报名所用手机号码，该手机号码将作为考生唯一识别标志，请务必认真填写，由于号码错误导致成绩无效，组委会不接受此类问题申诉。
8. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
   监考老师宣布开始后，你将有75分钟的时间完成测试。
9. All problems are provided with a Chinese translation for reference. In case of any discrepancy between the Chinese and English versions, the English version shall prevail.
   所有题目提供中文翻译作为参考。如果中英文两种版本之间存在差异，以英文版本为准。

1. The instructions on a 454-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 23 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

   一袋重量为 454 克的咖啡豆的使用说明上写着，冲泡一大杯手冲咖啡需要 23 克咖啡豆．问依照标准，用这袋咖啡豆最多可以冲泡多少杯大杯咖啡？

   (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

2. Jerry wrote down the ones digit of each of the first 2025 positive squares: $1, 4, 9, 6, 5, 6, \ldots$. What is the sum of all the numbers Jerry wrote down?

   Jerry 写下了前 2025 个正整数平方的个位数字：1，4，9，6，5，6，．．．．问 Jerry 写下的所有数的总和是多少？

   (A) 9025 **(B)** 9070 **(C)** 9090 **(D)** 9115 **(E)** 9160

3. A Pascal-like triangle has 10 as the top row and 10 followed by 1 as the second row. In each subsequent row the first number is 10, the last number is 1, and, as in the standard Pascal Triangle, each other number in the row is the sum of the two numbers directly above it. The first four rows are shown below. What is the sum of the digits of the sum of the numbers in the 12th row?

   在一个三角形数阵中，第一行为 10，第二行为 10 和 1．之后的每一行，第一个数是 10，最后一个数是 1，且如同标准的杨辉三角形，中间的其他每个数都是其上方两个数的和．下图显示了数阵的前四行．考虑第 12 行中所有数的总和，问这个总和的各位数字之和是多少？

   10
   10 1
   10 11 1
   10 21 12 1

   (A) 11 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 17

4. The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

   七进制的两位数 $\underline{a}\,\underline{b}$ 的值等于九进制的两位数 $\underline{b}\,\underline{a}$ 的值．问 $a+b$ 是多少？

   (A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 14

5. In $\triangle ABC$, $AB=10$, $AC=18$, and $\angle B=150^\circ$. Let O be the center of the circle containing points A, B, and C. What is the degree measure of $\angle CAO$?

   在 $\triangle ABC$ 中，$AB=10$，$AC=18$，且 $\angle B=150^\circ$．设 O 为经过 A，B，C 三点的圆的圆心．问 $\angle CAO$ 的度数是多少？

   (A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

6. The line $y=\frac{1}{3}x+1$ divides the square region defined by $0\le x\le 2$ and $0\le y\le 2$ into an upper region and a lower region. The line $x=a$ divides the upper region into two regions of equal area. Then a can be written as $s-\sqrt{t}$, where s and t are positive integers. What is $s+t$?

   直线 $y=\frac{1}{3}x+1$ 将由 $0\le x\le 2$ 和 $0\le y\le 2$ 定义的正方形区域分割成上方区域和下方区域．直线 $x=a$ 将上方区域分成两个面积相等的区域．已知 a 可以写成 $s-\sqrt{t}$ 的形式，其中 s 和 t 是正整数．问 $s+t$ 是多少？

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

7. Emmy says to Max, “I ordered 36 math club sweatshirts today.” Max asks, “How much did each shirt cost?” Emmy responds, “I'll give you a hint. The total cost was $\$\underline{A}\,\underline{B}\,\underline{B}.\underline{B}\,\underline{A}$, where A and B are digits and $A\neq 0$.” After a pause, Max says, “That was a good price.” What is the cost of each shirt in dollars? (Round to the nearest integer.)

   Emmy 对 Max 说：“我今天订了 36 件数学社的运动衫．”Max 问：“每件运动衫多少钱？”Emmy 回答说：“我给你一个提示．总成本是 $\$\underline{A}\,\underline{B}\,\underline{B}.\underline{B}\,\underline{A}$，其中 A 和 B 是数字且 $A\neq 0$．”停顿了一下后，Max 说：“这个价格很不错．”问每件运动衫的价格是多少美元？（四舍五入到整数．）

   (A) 11 **(B)** 14 **(C)** 15 **(D)** 17 **(E)** 18

8. Frances stands 12 meters directly south of a locked gate in a fence that runs east-west. Immediately behind the fence is a box of chocolates, located x meters east of the locked gate. An unlocked gate lies 10 meters east of the box, and another unlocked gate lies 9 meters west of the locked gate. Frances can reach the box by walking toward an unlocked gate, passing through it, and walking toward the box. It happens that the total distance Frances would travel would be the same via either unlocked gate. What is value of x?

一道东西走向的栅栏上有一扇上锁的门，Frances 站在上锁的门正南方 12 米处．在位于上锁的门以东 x 米处，紧贴着栅栏的后面有一盒巧克力．在巧克力盒以东 10 米处有一扇没锁的门，且在上锁的门以西 9 米处还有另一扇没锁的门．Frances 可以走向一扇没锁的门，穿过它，然后再走向巧克力盒．她通过任何一扇没锁的门走到巧克力盒的总路程是相同的．问 x 的值是多少？

(A) 4.5 **(B)** 5 **(C)** 5.5 **(D)** 6 **(E)** 6.5

9. How many ordered triples of integers (x, y, z) satisfy the following system of inequalities?

满足以下不等式组的三元有序整数组 (x, y, z) 有多少个？

$$\begin{aligned} -x-y-z &\le -2 \\ -x+y+z &\le 2 \\ x-y+z &\le 2 \\ x+y-z &\le 2 \end{aligned}$$

(A) 4 **(B)** 8 **(C)** 11 **(D)** 15 **(E)** 17

10. Let $f(n) = n^3 - 5n^2 + 2n + 8$, and let $g(n) = n^3 - 6n^2 + 5n + 12$. What is the sum of all integer values of n for which $\frac{f(n)}{g(n)}$ is also an integer?

设 $f(n) = n^3 - 5n^2 + 2n + 8$，且设 $g(n) = n^3 - 6n^2 + 5n + 12$．考虑所有使得 $\frac{f(n)}{g(n)}$ 也为整数的整数 n，问它们的总和是多少？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

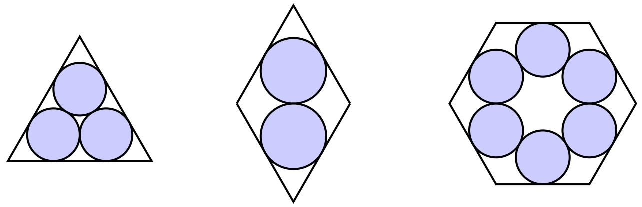
11. On Monday, 6 students went to the tutoring center at the same time, and each one was randomly assigned to one of the 6 tutors on duty. On Tuesday, the same 6 students showed up, the same 6 tutors were on duty, and the students were again randomly assigned to the tutors. What is the probability that exactly 2 students met with the same tutor both Monday and Tuesday?

星期一，有 6 名学生同时前往辅导中心，每名学生被随机分配给值班的 6 名辅导老师中的一位．星期二，这 6 名学生又去了，值班的还是这 6 名辅导老师，学生们再次被随机分配给这些辅导老师．问恰好有 2 名学生在星期一和星期二遇到相同的辅导老师的概率是多少？

(A) $\frac{1}{16}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{2}$

12. The figure below shows an equilateral triangle, a rhombus with a 60° angle, and a regular hexagon, each of them containing some mutually tangent congruent disks. Let T, R, and H, respectively, denote the ratio in each case of the total area of the disks to the area of the enclosing polygon. Which of the following is true?

下图展示了等边三角形，有一个角是 60° 的菱形，以及正六边形，其中每个图形都包含一些相互外切的全等圆盘．设 T，R，H 分别表示在每种情况下圆盘总面积与外框多边形的面积之比．问以下哪个选项是正确的？



(A) $H < R = T$ **(B)** $H = R < T$ **(C)** $H < R < T$

(D) $H < T < R$ **(E)** $T = R = H$

13. In a right triangle, the lengths of the two legs are in the ratio $1 : 2$. The altitude to the hypotenuse is divided into two segments of lengths $x < y$ by the median to the shorter leg of the triangle. What is the value of the ratio $\frac{x}{x+y}$?

一个直角三角形的两条直角边的长度之比是 $1:2$，斜边上的高被该三角形较短直角边上的中线分成两条线段，长度分别为 $x < y$. 问比值 $\frac{x}{x+y}$ 是多少?

(A) $\frac{3}{7}$ **(B)** $\frac{\sqrt{5}}{6}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{11}$ **(E)** $\frac{\sqrt{5}}{5}$

14. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

九名身高各不相同的运动员参加篮球队选拔. 他们依次从装有 3 条蓝色腕带，3 条红色腕带，3 条绿色腕带的袋子中随机抽取一条腕带，且抽取后不放回. 随后他们按腕带颜色被分成蓝队，红队，绿队. 每队中身高最高的成员被任命为队长. 问三名队长是三名最高的运动员的概率是多少?

(A) $\frac{2}{9}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{9}{28}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

15. The sum

$$\sum_{k=45}^{\infty} \frac{2025}{k^3 - k}$$

can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$?

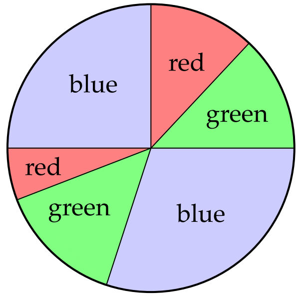
和式

$$\sum_{k=45}^{\infty} \frac{2025}{k^3 - k}$$

的计算结果可以表示为 $\frac{a}{b}$，其中 a 和 b 是互质的正整数. 问 $a + b$ 是多少?

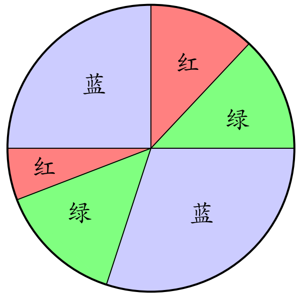
(A) 89 **(B)** 97 **(C)** 102 **(D)** 107 **(E)** 133

16. A circle has been divided into 6 sectors of 6 different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

一个圆被分成了 6 个大小不同的扇形. 然后将其中 2 个扇形涂成红色，2 个涂成绿色，2 个涂成蓝色，使得任意两个相邻的扇形颜色都不同. 下图展示了一种这样的着色方式.



问共有多少种不同的着色方式?

(A) 12 **(B)** 16 **(C)** 18 **(D)** 24 **(E)** 28

17. Consider a decreasing sequence of n positive integers $x_1 > x_2 > x_3 > \cdots > x_n$ that satisfies the following two conditions:

- The average (arithmetic mean) of the first 2 terms in the sequence is 2025.
- For all $3 \leq k \leq n$, the average of the first k terms in the sequence is 1 less than the average of the first $k-1$ terms in the sequence.

What is the greatest possible value of n,?

考虑由 n 个正整数组成的递减数列 $x_1 > x_2 > x_3 > \cdots > x_n$，它满足以下两个条件：

- 数列前 2 项的平均值（算术平均值）是 2025.
- 对于所有 $3 \leq k \leq n$，数列前 k 项的平均值比前 $k-1$ 项的平均值小 1.

问 n 的最大可能值是多少?

(A) 1013 **(B)** 1014 **(C)** 1016 **(D)** 2016 **(E)** 2025

18. What is the ones digit of the below expression

$$\lfloor\sqrt{1}\rfloor - \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor - \lfloor\sqrt{4}\rfloor + \lfloor\sqrt{5}\rfloor - \cdots - \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor\ ?$$

(Recall that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

下面算式计算结果的个位数字是多少?

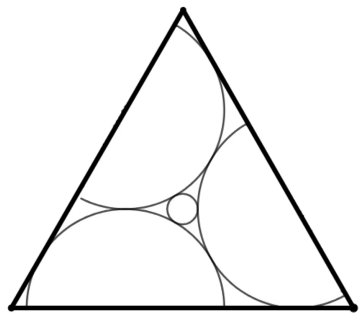
$$\lfloor\sqrt{1}\rfloor - \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor - \lfloor\sqrt{4}\rfloor + \lfloor\sqrt{5}\rfloor - \cdots - \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor\ ?$$

（注意：$\lfloor x \rfloor$ 表示不大于 x 的最大整数.）

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

19. Three congruent semicircles are inscribed in an equilateral triangle of side length 1 so that their diameters are on the sides of the triangle, adjacent semicircles are tangent to each other, and each semicircle is tangent to one side the triangle. A small circle centered at the center of the equilateral triangle is tangent to each of the three semicircles, as shown below. The diameter of the small circle can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers. What is $a+b+c$?

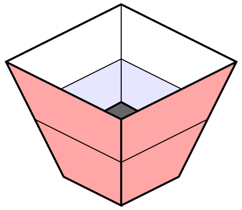
三个全等的半圆内接于边长为 1 的等边三角形中，它们的直径在三角形的边上，相邻的半圆彼此相切，且每个半圆与三角形的一条边相切. 如下图所示，一个小圆的圆心位于等边三角形的中心，并与这三个半圆都相切. 这个小圆的直径可以写成 $\frac{a-\sqrt{b}}{c}$ 的形式，其中 a, b, c 都是正整数. 问 $a+b+c$ 是多少?



(A) 5 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** 17

20. A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids. How many more minutes will it take to fill the remainder of the container?

如图所示，容器的底部是 1×1 的正方形，顶部是 3×3 的开放正方形，还有四个全等的梯形侧面．开始时容器是空的，用一根水管以恒定速率向容器中注水，需要35分钟才能将水注满至梯形侧面中线的高度．问还需要多少分钟才能将容器的剩余部分注满？



(A) 70 **(B)** 85 **(C)** 90 **(D)** 95 **(E)** 105

21. Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

对于 3×3 方格表中的 9 个方格，每个方格都要涂上红色、蓝色或黄色，且需满足以下条件：每个红色方格都与至少一个蓝色方格有公共边，每个蓝色方格都与至少一个黄色方格有公共边，每个黄色方格都与至少一个红色方格有公共边．通过旋转、反射或者它们的组合可以互相得到的涂色方式视为相同的，问共有多少种不同的涂色方式？

(A) 3 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

22. A seven-digit positive integer is chosen at random. What is the probability that the number is divisible by 7, given that the sum of its digits is 61?

随机选取一个七位正整数，已知其各位数字之和为 61，问这个数能被 7 整除的概率是多少？

(A) $\frac{1}{7}$ **(B)** $\frac{5}{28}$ **(C)** $\frac{3}{14}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{7}$

23. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

考虑有 141 行与 91 列的矩形方格表，每个方格中可填入两个数．Horace 与 Vera 分别将从 1 到 $141 \times 91 = 12{,}831$ 的所有数填入方格中．Horace 沿水平方向填写：他在第 1 行从左到右依序填入 1 至 91，在第 2 行从左到右依序填入 92 至 182，并以此类推直至填完第 141 行．Vera 沿垂直方向填写：她在第 1 列从上到下依序填入 1 至 141，在第 2 列从上到下依序填入 142 至 282，并以此类推直至填完第 91 列．问有多少个方格中两人填入的数相同？

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 19

24. A frog hops around the circle as shown below according to the following rules.

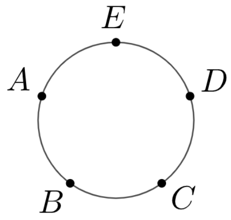
- It starts at A, and can move around only within positions A, B, C, D and E.
- If it is at A, B, C or D, then in the next step it moves to each of the two adjacent positions with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.
- If it reaches E, it will forever stay at that position.

What is the probability that the frog finally reaches E?

一只青蛙绕着下图所示的圆圈跳跃，其规则如下：

- 它从 A 出发，只在 A，B，C，D，E 这五个位置之间移动.
- 若它处于 A，B，C 或 D，它下一步移动到两个相邻位置中每一个的概率是 $\frac{1}{4}$，消失的概率是 $\frac{1}{2}$.
- 若它到达 E，则会永远停留在此位置.

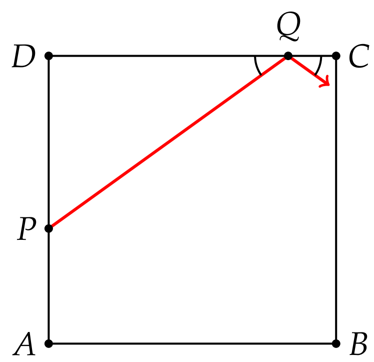
问青蛙最终能到达 E 的概率是多少？



(A) $\frac{1}{4}$ **(B)** $\frac{3}{11}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

25. Square $ABCD$ has sides of length 1. Points P and Q lie on $\overline{AD}$ and $\overline{CD}$, respectively, with $AP = \frac{2}{5}$ and $DQ = \frac{8}{11}$. A path begins along the line segment from P to Q and continues by reflecting against the sides of $ABCD$ (with congruent incoming and outgoing angles), as shown in the figure. If the path hits a vertex of the square, then it terminates there; otherwise it continues forever. How many times does this path reflect against the sides of $ABCD$?

正方形 $ABCD$ 的边长为 1，点 P 与点 Q 分别位于 $\overline{AD}$ 与 $\overline{CD}$ 上，且 $AP = \frac{2}{5}$，$DQ = \frac{8}{11}$. 如图所示，一条路径以线段 PQ 为起始，并在触及正方形 $ABCD$ 的边时反射前进（入射角与反射角相等）. 若路径碰到正方形的顶点，则在此终止；否则会继续下去，不断反射. 问这条路径在正方形 $ABCD$ 的边界上共反射多少次？



(A) 13 **(B)** 15 **(C)** 27

(D) 29 **(E)** Infinitely many ｜无穷多次