MAA AMC AMERICAN MATHEMATICS COMPETITION

MAA American Mathematics Competitions

76th Annual

AMC 12 B

Tuesday, November 12, 2024

INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
8. When you finish with the competition, please follow the directions of your competition manager.

The problems and solutions for this AMC 12 B were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

The MAA AMC office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this AMC 12 will be invited to take the 43rd annual American Invitational Mathematics Examination (AIME) on Thursday, February 6, 2025, or Wednesday, February 12, 2025. More details about the AIME can be found at maa.org/AMC.

1. In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

   (A) 2021 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 2025

2. What is $10! - 7! \cdot 6!$?

   (A) -120 **(B)** 0 **(C)** 120 **(D)** 600 **(E)** 720

3. For how many integer values of x is $|2x| \leq 7\pi$?

   (A) 16 **(B)** 17 **(C)** 19 **(D)** 20 **(E)** 21

4. Balls numbered $1, 2, 3, \ldots$ are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in bin B. The next 3 balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, balls numbered $22, 23, \ldots, 28$ are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

   (A) A **(B)** B **(C)** C **(D)** D **(E)** E

5. In the following expression, Melanie changed some of the plus signs to minus signs:

$$1 + 3 + 5 + 7 + \cdots + 97 + 99.$$

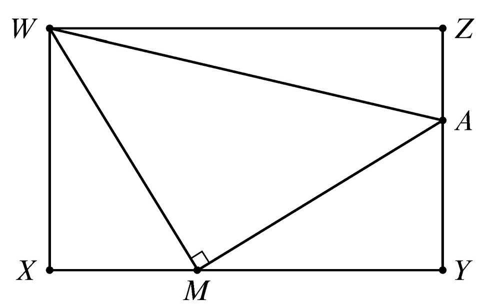
   When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

   (A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

6. The national debt of the United States is on track to reach 5×10^{13} dollars by 2033. How many digits does this number of dollars have when written as a numeral in base 5 ? (The approximation of $\log_{10} 5$ as 0.7 is sufficient for this problem.)

   (A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 26

7. In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies on $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of triangles $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?



   (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

8. What value of x satisfies
$$\frac{\log_2 x \cdot \log_3 x}{\log_2 x + \log_3 x} = 2\,?$$

(A) 25 **(B)** 32 **(C)** 36 **(D)** 42 **(E)** 48

9. A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x| + |y| \le 8$. A target T is the region where $\left(x^2 + y^2 - 25\right)^2 \le 49$. A dart is thrown and lands at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n} \cdot \pi$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

10. A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y, and z with $x \le y \le z$. The range of the list is 7, and the mean and the median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

11. Let $x_n = \sin^2(n^\circ)$. What is the mean of $x_1, x_2, x_3, \ldots, x_{90}$?

(A) $\frac{11}{45}$ **(B)** $\frac{22}{45}$ **(C)** $\frac{89}{180}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{91}{180}$

12. Suppose z is a complex number with positive imaginary part, with real part greater than 1, and with $|z| = 2$. In the complex plane, the four values 0, z, z^2, and z^3 are the vertices of a quadrilateral with area 15. What is the imaginary part of z ?

(A) $\frac{3}{4}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{5}{3}$

13. There are real numbers x, y, h, and k that satisfy the system of equations
$$x^2 + y^2 - 6x - 8y = h$$
$$x^2 + y^2 - 10x + 4y = k.$$

What is the minimum possible value of $h + k$?

(A) -54 **(B)** -46 **(C)** -34 **(D)** -16 **(E)** 16

14. How many different remainders can result when the 100th power of an integer is divided by 125 ?

(A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

15. A triangle in the coordinate plane has vertices $A(\log_2 1, \log_2 2)$, $B(\log_2 3, \log_2 4)$, and $C(\log_2 7, \log_2 8)$. What is the area of $\triangle ABC$?

(A) $\log_2 \frac{\sqrt{3}}{7}$ **(B)** $\log_2 \frac{3}{\sqrt{7}}$ **(C)** $\log_2 \frac{7}{\sqrt{3}}$ **(D)** $\log_2 \frac{11}{\sqrt{7}}$ **(E)** $\log_2 \frac{11}{\sqrt{3}}$

16. A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r ?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

17. Integers a and b are randomly chosen without replacement from the set of integers with absolute value not exceeding 10. What is the probability that the polynomial $x^3 + ax^2 + bx + 6$ has 3 distinct integer roots?

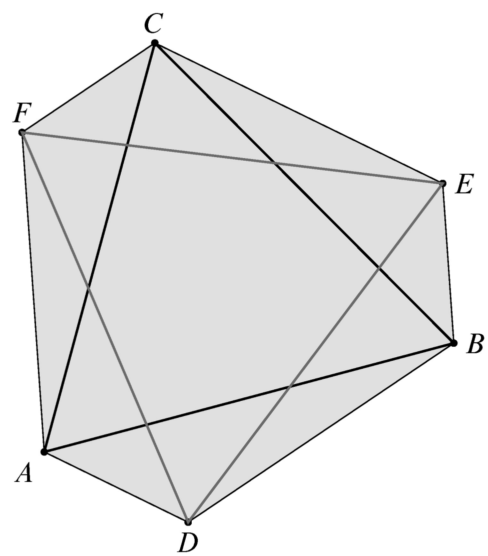
(A) $\frac{1}{240}$ **(B)** $\frac{1}{221}$ **(C)** $\frac{1}{105}$ **(D)** $\frac{1}{84}$ **(E)** $\frac{1}{63}$

18. The Fibonacci numbers are defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}} \text{ ?}$$

(A) 318 **(B)** 319 **(C)** 320 **(D)** 321 **(E)** 322

19. Equilateral $\triangle ABC$ with side length 14 is rotated about its center by angle θ, where $0 < \theta \leq 60°$, to form $\triangle DEF$. See the figure. The area of hexagon $ADBECF$ is $91\sqrt{3}$. What is $\tan\theta$?



(A) $\frac{3}{5}$ **(B)** $\frac{5\sqrt{3}}{11}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{11}{13}$ **(E)** $\frac{7\sqrt{3}}{13}$

20. Suppose A, B, and C are points in the plane with $AB = 40$ and $AC = 42$, and let x be the length of the line segment from A to the midpoint of $\overline{BC}$. Define a function f by letting $f(x)$ be the area of $\triangle ABC$. Then the domain of f is an open interval (p, q), and the maximum value r of $f(x)$ occurs at $x = s$. What is $p + q + r + s$?

(A) 909 **(B)** 910 **(C)** 911 **(D)** 912 **(E)** 913

21. The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are $3-4-5$ and $5-12-13$. What is the perimeter of the third triangle?

(A) 40 **(B)** 126 **(C)** 154 **(D)** 176 **(E)** 208

22. Let $\triangle ABC$ be a triangle with integer side lengths and the property that $\angle B = 2\angle A$. What is the least possible perimeter of such a triangle?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

23. A right pyramid has regular octagon $ABCDEFGH$ with side length 1 as its base and apex V. Segments $\overline{AV}$ and $\overline{DV}$ are perpendicular. What is the square of the height of the pyramid?

(A) 1 **(B)** $\dfrac{1+\sqrt{2}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\dfrac{3}{2}$ **(E)** $\dfrac{2+\sqrt{2}}{3}$

24. What is the number of ordered triples (a, b, c) of positive integers, with $a \le b \le c \le 9$, such that there exists a (non-degenerate) triangle $\triangle ABC$ with an integer inradius for which a, b, and c are the lengths of the altitudes from A to $\overline{BC}$, B to $\overline{AC}$, and C to $\overline{AB}$, respectively? (Recall that the inradius of a triangle is the radius of the largest possible circle that can be inscribed in the triangle.)

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

25. Pablo will decorate each of 6 identical white balls with either a striped or a dotted pattern, using either red or blue paint. He will decide on the color and pattern for each ball by flipping a fair coin for each of the 12 decisions he must make. After the paint dries, he will place the 6 balls in an urn. Frida will randomly select one ball from the urn and note its color and pattern. The events "the ball Frida selects is red" and "the ball Frida selects is striped" may or may not be independent, depending on the outcome of Pablo's coin flips. The probability that these two events are independent can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is m? (Recall that two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$.)

(A) 243 **(B)** 245 **(C)** 247 **(D)** 249 **(E)** 251