1. What is the value of $101 \cdot 9,901 - 99 \cdot 10,101$?

   $101 \cdot 9,901 - 99 \cdot 10,101$ 的值是多少？

   (A) 2 **(B)** 20 **(C)** 21 **(D)** 200 **(E)** 2020

2. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

   将 2024 写成若干个两位数的和，这些两位数不一定互不相同．问写出这样的和式最少需要多少个两位数？

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

3. A model used to estimate the time it will take to hike to the top of a mountain on a trail is of the form $T = aL + bG$, where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

   用于估算沿着山间小径登顶所需时间的模型表示为 $T = aL + bG$，其中 a 和 b 是常数，T 是以分钟为单位计的时间，L 是以英里为单位计的小径长度，G 是以英尺为单位计的海拔增加量．该模型估算，沿着一条长 1.5 英里且海拔提升 800 英尺的小径需要 69 分钟登顶，沿着一条长 1.2 英里且海拔提升 1100 英尺的小径也同样需要 69 分钟登顶．根据该模型，沿着一条长 4.2 英里且海拔提升 4000 英尺的小径需要多少分钟才能登顶？

   (A) 240 **(B)** 246 **(C)** 252 **(D)** 258 **(E)** 264

4. Let n be the least prime number that can be written as the sum of 5 distinct prime numbers. What is the sum of the digits of n ?

   设 n 是能够表示成 5 个不同质数之和的最小质数．问 n 的各位数字之和是多少？

   (A) 5 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 11

5. What is the least value of n such that $n!$ is a multiple of 2024 ?

   使得 $n!$ 是 2024 的倍数的最小正整数 n 是多少？

   (A) 11 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 253

6. What is the minimum number of successive swaps of adjacent letters in the string ABCDEF that are needed to change the string to FEDCBA? (For example, 3 swaps are required to change ABC to CBA; one such sequence of swaps is ABC $\to$ BAC $\to$ BCA $\to$ CBA.)

   将字符串 ABCDEF 变成 FEDCBA，最少需要多少次相邻字母的交换？（例如，将 ABC 变成 CBA 需要 3 次交换；一个这样的交换序列是 ABC $\to$ BAC $\to$ BCA $\to$ CBA．）

   (A) 6 **(B)** 10 **(C)** 12 **(D)** 15 **(E)** 24

7. Amy, Bomani, Charlie, and Daria work in a chocolate factory. On Monday Amy, Bomani, and Charlie started working at 1:00 PM and were able to pack 4, 3, and 3 packages, respectively, every 3 minutes. At some later time, Daria joined the group, and Daria was able to pack 5 packages every 4 minutes. Together, they finished packing 450 packages at exactly 2:45 PM. At what time did Daria join the group?

   Amy，Bomani，Charlie，Daria 在一家巧克力工厂工作．星期一，Amy，Bomani，Charlie 从 1:00 PM 开始工作，他们每 3 分钟能完成封装的包裹数分别是 4 个，3 个，3 个．在之后的某个时间，Daria 加入了工作，她每 4 分钟能封装 5 个包裹．他们一起在 2:45 PM 准时完成了 450 个包裹的封装．问 Daria 是何时加入工作的？

   (A) 1:25 PM **(B)** 1:35 PM **(C)** 1:45 PM **(D)** 1:55 PM **(E)** 2:05 PM

8. The product of three integers is 60. What is the least possible positive sum of the three integers?

三个整数的乘积是 60. 如果它们的和是正数，那么这个正数最小可能是多少?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 13

9. In how many ways can 6 juniors and 6 seniors form 3 disjoint teams of 4 people so that each team has 2 juniors and 2 seniors?

6 名大三学生和 6 名大四学生要组成 3 个互不重叠的 4 人小组，每个小组必须包含 2 名大三学生和 2 名大四学生，问共有多少种不同的分组方式?

(A) 720 **(B)** 1350 **(C)** 2700 **(D)** 3280 **(E)** 8100

10. How many ordered pairs of integers (m,n) satisfy $\sqrt{n^2-49}=m$?

满足 $\sqrt{n^2-49}=m$ 的有序整数对 (m,n) 有多少个?

(A) 1 **(B)** 2 **(C)** 3

(D) 4 **(E)** infinitely many | 无穷多个

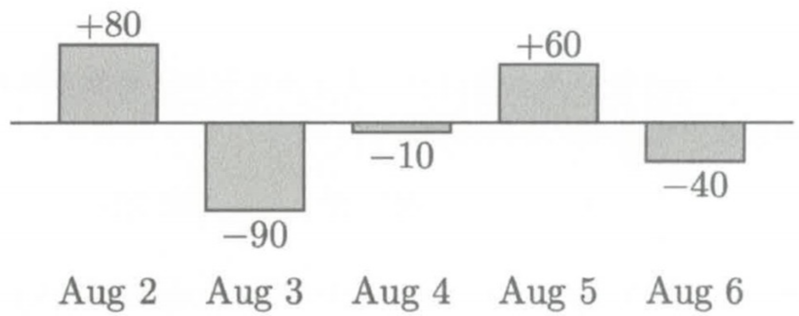
11. Consider the following operation. Given a positive integer n, if n is a multiple of 3 , then you replace n by $\frac{n}{3}$. If n is not a multiple of 3 , then you replace n by $n+10$. Then continue this process. For example, beginning with $n=4$, this procedure gives $4\to 14\to 24\to 8\to 18\to 6\to 2\to 12\to\cdots$. Suppose you start with $n=100$. What value results if you perform this operation exactly 100 times?

考虑以下的操作. 给定一个正整数 n，如果 n 是 3 的倍数，则将 n 替换为 $\frac{n}{3}$；如果 n 不是 3 的倍数，则将 n 替换为 $n+10$. 然后不断进行这个操作. 例如，从 $n=4$ 开始，这个操作将给出 $4\to 14\to 24\to 8\to 18\to 6\to 2\to 12\to\cdots$. 假设从 $n=100$ 开始，执行这个操作恰好 100 次后将得到什么数值?

(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

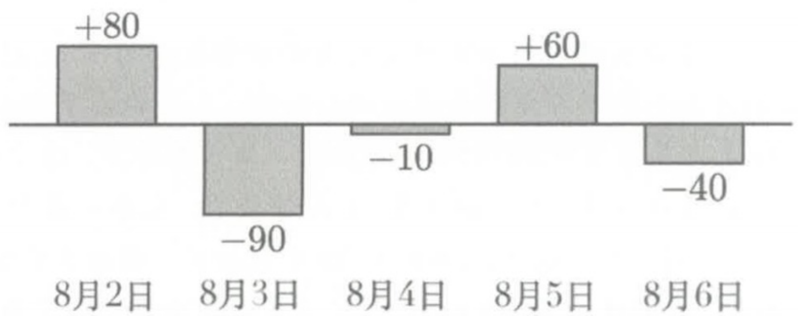
12. Zelda played the Adventures of Math game on August 1 and scored 1700 points. She continued to play daily over the next 5 days. The bar chart below shows the daily change in her score compared to the day before. (For example, Zelda's score on August 2 was $1700+80=1780$ points.) What was Zelda's average score in points over the 6 days?

Daily Change in Score from August 2 to 6



Zelda 在 8 月 1 日玩“数学冒险”游戏，积分是 1700 分. 在接下来的 5 天里她每天都继续玩. 下面的柱状图显示了她每天相比前一天的积分变化. (例如，Zelda 在 8 月 2 日的积分是 $1700+80=1780$ 分.) 问在这 6 天中她的平均积分是多少分?

从8月2日到8月6日的每日积分变化



(A) 1700 **(B)** 1702 **(C)** 1703 **(D)** 1713 **(E)** 1715

13. Two transformations are said to *commute* if applying the first followed by the second gives the same result as applying the second followed by the first. Consider these four transformations of the coordinate plane:

- a translation 2 units to the right,
- a 90°-rotation counterclockwise about the origin,
- a reflection across the x-axis, and
- a dilation centered at the origin with scale factor 2.

Of the 6 pairs of distinct transformations from this list, how many commute?

两个变换被称作是"可交换的",如果先应用第一个变换,再应用第二个变换得到的结果,与先应用第二个变换,再应用第一个变换得到的结果相同.考虑以下坐标平面的四个变换:

- 向右平移 2 个单位,
- 绕原点逆时针旋转 90°,
- 关于 x 轴的反射,
- 以原点为中心,比例因子为 2 的放大.

这个列表中任意两个不同的变换可以组成 6 对.问其中有多少对是可交换的?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

设 M 是使得 $M + 1213$ 和 $M + 3773$ 都是完全平方数的最大整数.问 M 的个位数字是几?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

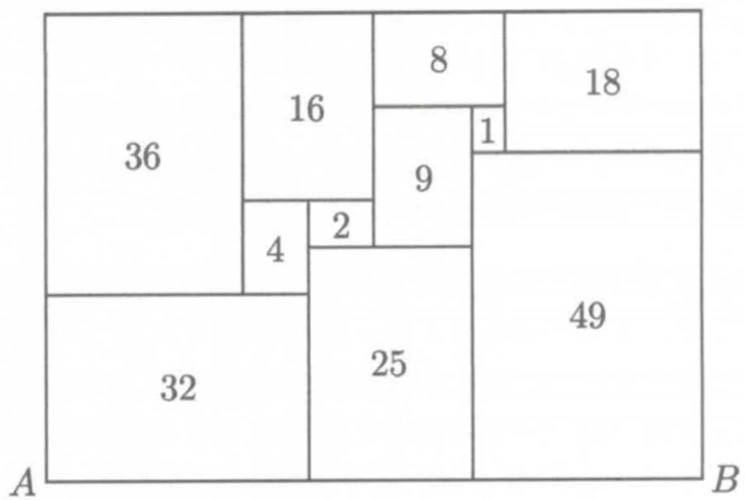
15. One side of an equilateral triangle of height 24 lies on line ℓ. A circle of radius 12 is tangent to ℓ and is externally tangent to the triangle. The area of the region exterior to the triangle and the circle and bounded by the triangle, the circle, and line ℓ can be written as $a\sqrt{b} - c\pi$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is $a + b + c$?

高为 24 的等边三角形的一条边位于直线 ℓ 上.半径为 12 的圆与直线 ℓ 相切,并且与三角形外切.在三角形和圆的外部,由三角形,圆,直线 ℓ 所围成的区域的面积可以表示为 $a\sqrt{b} - c\pi$,其中 a,b,c 是正整数,且 b 不能被任何质数的平方整除.问 $a + b + c$ 是多少?

(A) 72 **(B)** 73 **(C)** 74 **(D)** 75 **(E)** 76

16. All of the rectangles in the figure below, which is drawn to scale, are similar to the enclosing rectangle. Each number represents the area of its rectangle. What is length AB?

在按比例绘制的下图中,所有矩形都与整个的大矩形相似.每个数表示其所在矩形的面积.问 AB 的长度是多少?



(A) $4 + 4\sqrt{5}$ **(B)** $10\sqrt{2}$ **(C)** $5 + 5\sqrt{5}$ **(D)** $10\sqrt[4]{8}$ **(E)** 20

17. There are exactly K positive integers b with $5 \le b \le 2024$ such that the base-b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K ?

满足 $5 \le b \le 2024$，且 b 进制数 2024_b 能被 16（16 是以十进制表示）整除的正整数 b 恰好有 K 个．问 K 的各位数字之和是多少？

(A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

18. The first three terms of a geometric sequence are the integers a, 720, and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b ?

一个等比数列的前三项是整数 a，720，b，并且 $a < 720 < b$．问 b 的最小可能值的各位数字之和是多少？

(A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

19. Two teams are in a best-two-out-of-three playoff: the teams will play at most 3 games, and the winner of the playoff is the first team to win 2 games. The first game is played on Team A's home field, and the remaining games are played on Team B's home field. Team A has a $\frac{2}{3}$ chance of winning at home, and its probability of winning when playing away from home is p. Outcomes of the games are independent. The probability that Team A wins the playoff is $\frac{1}{2}$. Then p can be written in the form $\frac{1}{2}(m - \sqrt{n})$, where m and n are positive integers. What is $m + n$?

两个队伍进行三局两胜制的季后赛：即两队之间最多进行 3 场比赛，首先赢得 2 场比赛的队伍获胜．第一场比赛在 A 队的主场进行，剩余的比赛在 B 队的主场进行．A 队在主场获胜的概率是 $\frac{2}{3}$，在客场获胜的概率是 p．每场比赛的结果都是相互独立的．A 队赢得季后赛的概率是 $\frac{1}{2}$．已知 p 可以表示为 $\frac{1}{2}(m - \sqrt{n})$ 的形式，其中 m 和 n 是正整数．问 $m + n$ 是多少？

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

20. Let S be a subset of $\{1, 2, 3, \ldots, 2024\}$ such that the following two conditions hold:

- If x and y are distinct elements of S, then $|x - y| > 2$.
- If x and y are distinct odd elements of S, then $|x - y| > 6$.

What is the maximum possible number of elements in S ?

设 S 是 $\{1, 2, 3, \ldots, 2024\}$ 的子集，满足以下两个条件：

- 若 x 和 y 是 S 中的不同元素，则 $|x - y| > 2$.
- 若 x 和 y 是 S 中的不同奇数元素，则 $|x - y| > 6$.

问 S 中最多可能有多少个元素？

(A) 436 **(B)** 506 **(C)** 608 **(D)** 654 **(E)** 675

21. The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5 . The numbers in positions $(5,5)$, $(2,4)$, $(4,3)$, and $(3,1)$ are 0, 48, 16, and 12, respectively. What number is in position $(1,2)$?

在 5×5 的整数数表中，每行的数按其现有顺序组成一个五项的等差数列，每列的数也按其现有顺序组成一个五项的等差数列．已知在位置 $(5,5)$，$(2,4)$，$(4,3)$，$(3,1)$ 处的数分别是 0，48，16，12．问位置 $(1,2)$ 处的数是多少？

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19 **(B)** 24 **(C)** 29 **(D)** 34 **(E)** 39

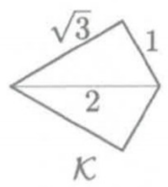
22. Integers $a, b,$ and c satisfy $ab + c = 100, bc + a = 87,$ and $ca + b = 60$. What is $ab + bc + ca$?

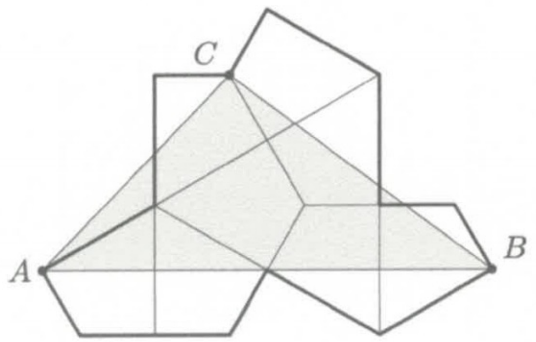
整数 a，b，c 满足 $ab+c=100$，$bc+a=87$，$ca+b=60$. 问 $ab+bc+ca$ 是多少？

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

23. Let $\mathcal{K}$ be the kite formed by joining two right triangles with legs 1 and $\sqrt{3}$ along a common hypotenuse. Eight copies of $\mathcal{K}$ are used to form the polygon shown below. What is the area of triangle $\triangle ABC$?

设 $\mathcal{K}$ 是由两个直角三角形沿着公共斜边连接而成的筝形，这两个直角三角形的直角边长度分别为 1 和 $\sqrt{3}$. 用 8 个 $\mathcal{K}$ 可以组成如下图所示的多边形. 求三角形 $\triangle ABC$ 的面积.





(A) $2+3\sqrt{3}$ **(B)** $\frac{9}{2}\sqrt{3}$ **(C)** $\frac{10+8\sqrt{3}}{3}$ **(D)** 8 **(E)** $5\sqrt{3}$

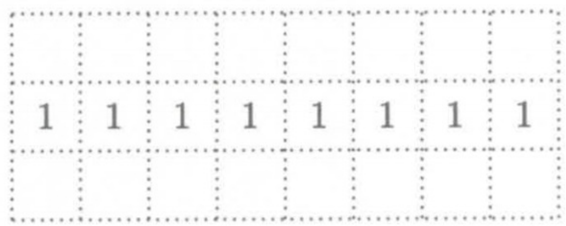
24. A bee is moving in three-dimensional space. A fair six-sided die with faces labeled A^+, A^-, B^+, B^-, C^+, and C^- is rolled. Suppose the bee occupies the point (a,b,c). If the die shows A^+, then the bee moves to the point $(a+1,b,c)$, and if the die shows A^-, then the bee moves to the point $(a-1,b,c)$. Analogous moves are made with the other four outcomes. Suppose the bee starts at the point $(0,0,0)$ and the die is rolled four times. What is the probability that the bee traverses four distinct edges of some unit cube?

一只蜜蜂在三维空间中移动. 一个均匀的六面骰子的各个面分别标记为 A^+，A^-，B^+，B^-，C^+，C^-. 假设蜜蜂位于点 (a,b,c)，如果骰子显示 A^+，蜜蜂就移动到点 $(a+1,b,c)$；如果显示 A^-，就移动到点 $(a-1,b,c)$. 其他四种情况也以类似的方式移动. 假设蜜蜂从点 $(0,0,0)$ 开始，骰子被投掷四次. 问蜜蜂恰好经过某个单位立方体的四条不同边的概率是多少？

(A) $\frac{1}{54}$ **(B)** $\frac{7}{54}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{5}{18}$ **(E)** $\frac{2}{5}$

25. The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?

下图显示了由 1 英寸 × 1 英寸的方格组成的，宽是 8 个方格，高是 3 个方格的虚线网格. Carl 要沿着一些方格的边放置长为 1 英寸的牙签，以形成一个不自相交的闭合折线. 方格中的数表示该方格被牙签覆盖的边的个数；对于没有数的方格，其被牙签覆盖的边的个数没有限制. 问 Carl 有多少种不同的放置牙签的方法？



(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196