MAA AMC AMERICAN MATHEMATICS COMPETITION

MAA American Mathematics Competitions

26th Annual

AMC 10 B

Tuesday, November 12, 2024

INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
8. When you finish with the competition, please follow the directions of your competition manager.

The problems and solutions for this AMC 10 B were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

The MAA AMC office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this AMC 10 will be invited to take the 43rd annual American Invitational Mathematics Examination (AIME) on Thursday, February 6, 2025, or Wednesday, February 12, 2025. More details about the AIME can be found at maa.org/AMC.

1. In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

   (A) 2021 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 2025

2. What is $10! - 7! \cdot 6!$?

   (A) -120 **(B)** 0 **(C)** 120 **(D)** 600 **(E)** 720

3. For how many integer values of x is $|2x| \leq 7\pi$?

   (A) 16 **(B)** 17 **(C)** 19 **(D)** 20 **(E)** 21

4. Balls numbered $1, 2, 3, \ldots$ are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in bin B. The next 3 balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, balls numbered $22, 23, \ldots, 28$ are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

   (A) A **(B)** B **(C)** C **(D)** D **(E)** E

5. In the following expression, Melanie changed some of the plus signs to minus signs:

   $$1 + 3 + 5 + 7 + \cdots + 97 + 99.$$

   When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

   (A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

6. A rectangle has integer length sides and an area of 2024. What is the least possible perimeter of the rectangle?

   (A) 160 **(B)** 180 **(C)** 222 **(D)** 228 **(E)** 390

7. What is the remainder when $7^{2024} + 7^{2025} + 7^{2026}$ is divided by 19 ?

   (A) 0 **(B)** 1 **(C)** 7 **(D)** 11 **(E)** 18

8. Let N be the product of all the positive integer divisors of 42. What is the units digit of N ?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

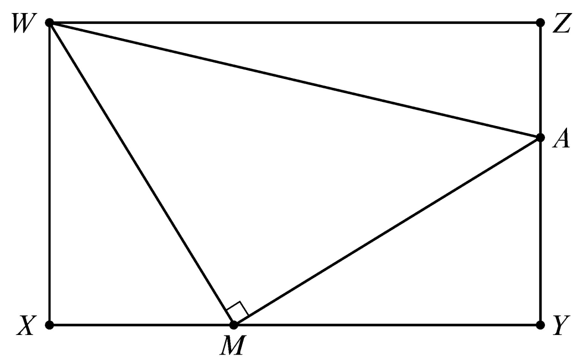
9. Real numbers a, b, and c have arithmetic mean 0. The arithmetic mean of a^2, b^2, and c^2 is 10. What is the arithmetic mean of ab, ac, and bc ?

(A) -5 **(B)** $-\frac{10}{3}$ **(C)** $-\frac{10}{9}$ **(D)** 0 **(E)** $\frac{10}{9}$

10. Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side $\overline{AD}$. Let F be the intersection of lines EB and AC. What is the ratio of the area of quadrilateral $CDEF$ to the area of $\triangle CFB$?

(A) $5:4$ **(B)** $4:3$ **(C)** $3:2$ **(D)** $5:3$ **(E)** $2:1$

11. In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies on $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of triangles $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?



(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

12. A group of 100 students from different countries meet at a mathematics competition. Each student speaks the same number of languages, and, for every pair of students A and B, student A speaks some language that student B does not speak, and student B speaks some language that student A does not speak. What is the least possible total number of languages spoken by all the students?

(A) 9 **(B)** 10 **(C)** 12 **(D)** 51 **(E)** 100

13. Positive integers x and y satisfy the equation $\sqrt{x} + \sqrt{y} = \sqrt{1183}$. What is the minimum possible value of $x + y$?

(A) 585 **(B)** 595 **(C)** 623 **(D)** 700 **(E)** 791

14. A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x| + |y| \le 8$. A target T is the region where $\left(x^2 + y^2 - 25\right)^2 \le 49$. A dart is thrown and lands at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n} \cdot \pi$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

15. A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y, and z with $x \leq y \leq z$. The range of the list is 7, and the mean and the median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

16. Jerry likes to play with numbers. One day, he wrote all the integers from 1 to 2024 on the whiteboard. Then he repeatedly chose four numbers on the whiteboard, erased them, and replaced them by either their sum or their product. (For example, Jerry's first step might have been to erase 1, 2, 3, and 5, and then write either 11, their sum, or 30, their product, on the whiteboard.) After repeatedly performing this operation, Jerry noticed that all the remaining numbers on the whiteboard were odd. What is the maximum possible number of integers on the whiteboard at that time?

(A) 1010 **(B)** 1011 **(C)** 1012 **(D)** 1013 **(E)** 1014

17. In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result of the race might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second; and Bruna is fifth. How many different results of the race are possible?

(A) 180 **(B)** 361 **(C)** 420 **(D)** 431 **(E)** 720

18. How many different remainders can result when the 100th power of an integer is divided by 125 ?

(A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

19. In the following table, each question mark is to be replaced by "Possible" or "Not Possible" to indicate whether a nonvertical line with the given slope can contain the given number of lattice points (points both of whose coordinates are integers). How many of the 12 entries will be "Possible"?

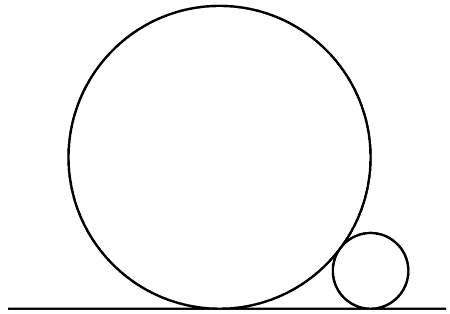
	zero	**exactly one**	**exactly two**	**more than two**
zero slope	**?**	**?**	**?**	**?**
nonzero rational slope	**?**	**?**	**?**	**?**
irrational slope	**?**	**?**	**?**	**?**

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

20. Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe from a different pair. In how many ways can these six shoes be lined up?

(A) 60 **(B)** 72 **(C)** 90 **(D)** 108 **(E)** 120

21. Two straight pipes (circular cylinders), with radii 1 and $\frac{1}{4}$, lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?



(A) $\frac{1}{9}$ **(B)** 1 **(C)** $\frac{10}{9}$ **(D)** $\frac{11}{9}$ **(E)** $\frac{19}{9}$

22. A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r ?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

23. The Fibonacci numbers are defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}\ ?$$

(A) 318 **(B)** 319 **(C)** 320 **(D)** 321 **(E)** 322

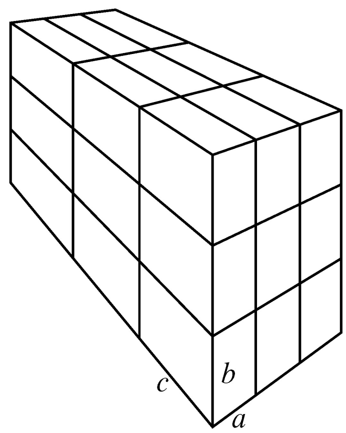
24. Let

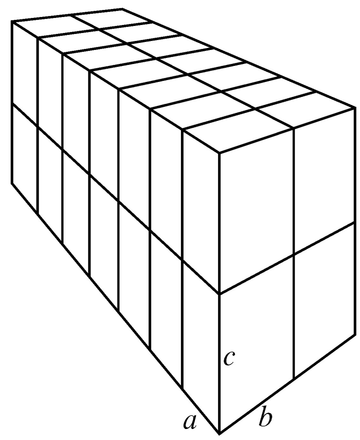
$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}.$$

How many of the values $P(2022)$, $P(2023)$, $P(2024)$, and $P(2025)$ are integers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

25. Each of 27 bricks (right rectangular prisms) has dimensions $a \times b \times c$, where a, b, and c are pairwise relatively prime positive integers. These bricks are arranged to form a $3 \times 3 \times 3$ block, as shown on the left below. A 28th brick with the same dimensions is introduced, and these bricks are reconfigured into a $2 \times 2 \times 7$ block, shown on the right. The new block is 1 unit taller, 1 unit wider, and 1 unit deeper than the old one. What is $a + b + c$?





(A) 88 **(B)** 89 **(C)** 90 **(D)** 91 **(E)** 92