1. Every morning Aya goes for a 9-kilometer-long walk and stops at a coffee shop afterwards. When she walks at a constant speed of s kilometers per hour, the walk takes her 4 hours, including t minutes spent in the coffee shop. When she walks at $s+2$ kilometers per hour, the walk takes her 2 hours and 24 minutes, including t minutes spent in the coffee shop. Suppose Aya walks at $s+\frac{1}{2}$ kilometers per hour. Find the number of minutes the walk takes her, including the t minutes spent in the coffee shop.

   每天早上，Aya 步行 9 千米，然后在咖啡店停留。当她以每小时 s 千米的恒定速度行走时，整个行程需要 4 小时，包括在咖啡店停留的 t 分钟。当她以每小时 $s+2$ 千米的速度行走时，整个行程需要 2 小时 24 分钟，包括在咖啡店停留的 t 分钟。假设 Aya 以每小时 $s+\frac{1}{2}$ 千米的速度行走，那么整个行程需要多少分钟，包括在咖啡店停留的 t 分钟?

   Answer | 答案: $\boxed{204}$

2. There exist real numbers x and y, both greater than 1, such that

$$\log_x\left(y^x\right)=\log_y\left(x^{4y}\right)=10.$$

   Find xy.

   存在大于 1 的实数 x 和 y，满足

$$\log_x\left(y^x\right)=\log_y\left(x^{4y}\right)=10。$$

   求 xy。

   Answer | 答案: $\boxed{025}$

3. Alice and Bob play the following game. A stack of n tokens lies before them. The players take turns with Alice going first. On each turn, the player removes either 1 token or 4 tokens from the stack. Whoever removes the last token wins. Find the number of positive integers n less than or equal to 2024 for which there exists a strategy for Bob that guarantees that Bob will win the game regardless of Alice's play.

Alice 和 Bob 玩下述的游戏。他们面前有摆成一堆的 n 个筹码。玩家轮流操作，Alice 先开始。每一次，玩家可以从堆中取走 1 个或 4 个筹码。取走最后一个筹码的人获胜。在小于或等于 2024 的正整数中，有多少个数可以作为 n，使得 Bob 存在一种策略，无论 Alice 如何行动，都能确保自己获胜?

Answer | 答案: $\boxed{809}$

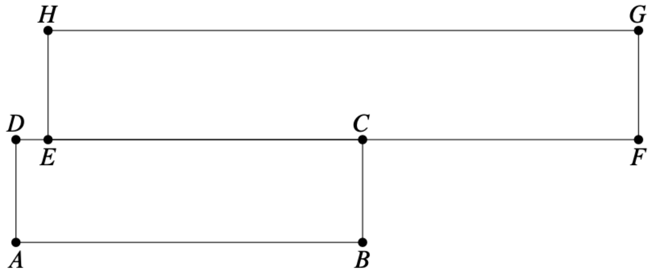
4. Jen enters a lottery by selecting four distinct elements of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then four distinct elements of S are drawn at random. Jen wins a prize if at least two of her numbers are drawn, and she wins the grand prize if all four of her numbers are drawn. The probability that Jen wins the grand prize given that Jen wins a prize is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Jen 通过从集合 $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ 中选择四个不同的元素参加抽奖。然后四个不同的元素被从集合 S 中随机抽取。如果 Jen 选择的数至少有两个被抽取，那么她就获奖了；如果 Jen 选择的四个数都被抽取了，那么她将赢得大奖。在 Jen 获奖的前提下，她赢得大奖的概率为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数。求 $m + n$。

Answer | 答案: $\boxed{116}$

5. Rectangle $ABCD$ has dimensions $AB = 107$ and $BC = 16$, and rectangle $EFGH$ has dimensions $EF = 184$ and $FG = 17$. Points D, E, C, and F lie on line DF in that order, and A and H lie on opposite sides of line DF, as shown. Points A, D, H, and G lie on a common circle. Find CE.

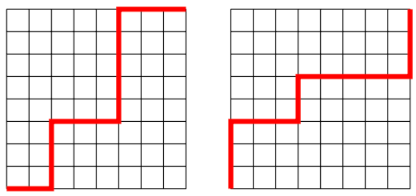
在矩形 $ABCD$ 中，$AB = 107$，$BC = 16$，在矩形 $EFGH$ 中，$EF = 184$，$FG = 17$。如图所示，点 D，E，C，F 按此顺序位于直线 DF 上，点 A 和 H 位于直线 DF 的两侧。点 A，D，H，G 位于同一个圆上。求 CE。



Answer | 答案: 104

6. Consider the paths of length 16 that follow the lines from the lower left corner to the upper right corner on an 8×8 grid. Find the number of such paths that change direction exactly four times, as in the examples shown below.

考虑在 8×8 的方格表中，从左下角到右上角的长度为 16 的路径。求其中行进方向恰好改变四次的路径的数目，下图给出了这样的例子。



Answer | 答案: 294

7. Find the greatest possible real part of

$$(75 + 117i)z + \frac{96 + 144i}{z},$$

where z is a complex number with $|z| = 4$. Here $i = \sqrt{-1}$.

设 z 是一个复数，满足 $|z| = 4$，求

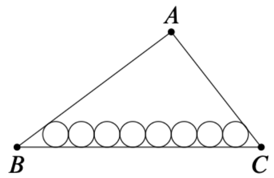
$$(75 + 117i)z + \frac{96 + 144i}{z}$$

实部的最大可能值。这里 $i = \sqrt{-1}$。

Answer | 答案: 540

8. Eight circles of radius 34 can be placed tangent to side $\overline{BC}$ of $\triangle ABC$ so that the circles are sequentially tangent to each other, with the first circle being tangent to $\overline{AB}$ and the last circle being tangent to $\overline{AC}$, as shown. Similarly, 2024 circles of radius 1 can be placed tangent to $\overline{BC}$ in the same manner. The inradius of $\triangle ABC$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

如图所示，可以放置 8 个半径为 34 的圆与 $\triangle ABC$ 的边 $\overline{BC}$ 相切，使得相邻的两个圆都互相外切，第一个圆与边 $\overline{AB}$ 相切，最后一个圆与边 $\overline{AC}$ 相切。类似的，以相同的方式的可以放置 2024 个半径为 1 的圆与边 $\overline{BC}$ 相切。$\triangle ABC$ 的内切圆半径可以表示为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数。求 $m+n$。



Answer | 答案: $\boxed{197}$

9. Let $A, B, C,$ and D be points on the hyperbola $\frac{x^2}{20} - \frac{y^2}{24} = 1$ such that $ABCD$ is a rhombus whose diagonals intersect at the origin. Find the greatest real number that is less than BD^2 for all such rhombi.

点 A，B，C，D 在双曲线 $\frac{x^2}{20} - \frac{y^2}{24} = 1$ 上，使得 $ABCD$ 是对角线交于原点的菱形。求最大的实数，使得对于所有这样的菱形，它小于 BD^2。

Answer | 答案: $\boxed{480}$

10. Let $\triangle ABC$ have side lengths $AB = 5$, $BC = 9$, and $CA = 10$. The tangents to the circumcircle of $\triangle ABC$ at B and C intersect at point D, and $\overline{AD}$ intersects the circumcircle at $P \neq A$. The length of $\overline{AP}$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

$\triangle ABC$ 的三边长是 $AB = 5$，$BC = 9$，$CA = 10$。过 B 和 C 的 $\triangle ABC$ 的外接圆的切线相交于点 D，直线 $\overline{AD}$ 与该圆交于点 P（$P \neq A$）。$\overline{AP}$ 的长度可以表示为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数。求 $m + n$。

Answer | 答案: $\boxed{113}$

11. Each vertex of a regular octagon is independently colored either red or blue with equal probability. The probability that the octagon can then be rotated so that all of the blue vertices move to positions where there had been red vertices is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

正八边形的每个顶点都独立等概率的涂成红色或蓝色，所有蓝色的顶点可以通过旋转正八边形使得它们移动到原来红色顶点的位置的概率为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数。求 $m + n$。

Answer | 答案: $\boxed{371}$

12. Define $f(x) = \left||x| - \frac{1}{2}\right|$ and $g(x) = \left||x| - \frac{1}{4}\right|$. Find the number of intersections of the graphs of

$$y = 4g(f(\sin(2\pi x))) \quad \text{and} \quad x = 4g(f(\cos(3\pi y))).$$

定义 $f(x) = \left||x| - \frac{1}{2}\right|$ 和 $g(x) = \left||x| - \frac{1}{4}\right|$。求

$$y = 4g(f(\sin(2\pi x))) \quad 和 \quad x = 4g(f(\cos(3\pi y)))$$

图像的交点的个数。

Answer | 答案: 385

13. Let p be the least prime number for which there exists an integer n such that $n^4 + 1$ is divisible by p^2. Find the least positive integer m such that $m^4 + 1$ is divisible by p^2.

设 p 是最小的质数，具有性质：存在整数 n，使得 $n^4 + 1$ 可被 p^2 整除。求最小的正整数 m，使得 $m^4 + 1$ 可被 p^2 整除。

Answer | 答案: 110

14. Let $ABCD$ be a tetrahedron such that $AB = CD = \sqrt{41}$, $AC = BD = \sqrt{80}$, and $BC = AD = \sqrt{89}$. There exists a point I inside the tetrahedron such that the distances from I to each of the faces of the tetrahedron are all equal. This distance can be written in the form $\frac{m\sqrt{n}}{p}$, where m, n, and p are positive integers, m and p are relatively prime, and n is not divisible by the square of any prime. Find $m + n + p$.

在四面体 $ABCD$ 中，$AB = CD = \sqrt{41}$，$AC = BD = \sqrt{80}$，$BC = AD = \sqrt{89}$。在四面体的内部存在一个点 I，使得 I 到四面体的每个面的距离都相等。这个距离可以表示为 $\frac{m\sqrt{n}}{p}$，其中 m，n，p 是正整数，m 和 p 互质，n 不可被任何质数的平方整除。求 $m + n + p$。

Answer | 答案: 104

15. Let $\mathcal{B}$ be the set of rectangular boxes with surface area 54 and volume 23. Let r be the radius of the smallest sphere that can contain each of the rectangular boxes that are elements of $\mathcal{B}$. The value of r^2 can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p+q$.

设 $\mathcal{B}$ 是表面积为 54、体积为 23 的所有长方体构成的集合。设能够包含 $\mathcal{B}$ 中每个长方体的最小球的半径是 r。r^2 可以表示为 $\frac{p}{q}$，其中 p 和 q 是互质的正整数。求 $p+q$。

Answer | 答案: $\boxed{721}$