2017 AMC12B

Problem 1

Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

Kymbrea 的漫画书集目前有 30 本漫画书，并且她现在还在以每个月 2 本漫画书的速度向她的漫画书集中增添新书。Lashawn 的漫画书集目前有 10 本漫画书，并且目前他在以每个月 6 本漫画书的速度向他的漫画书集中增添新书。问经过多少个月 Lashawn 的漫画书是 Kymbrea 书的 2 倍?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Problem 2

Real numbers x, y, and z satisfy the inequalities $0<x<1$, $-1<y<0$, and $1<z<2$. Which of the following numbers is necessarily positive?

实数 x，y，z 满足不等式 $0<x<1$，$-1<y<0$，$1<z<2$，下面哪个数字是正数?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

Problem 3

Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y}=-2$. What is the value of $\frac{x+3y}{3x-y}$?

假设 x 和 y 是非零实数，满足$\frac{3x+y}{x-3y}=-2$，则$\frac{x+3y}{3x-y}$的值为多少？

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Problem 4

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

Samia 骑着自行车去拜访她的朋友，平均速度为 17 公里每小时，当她走了一半的路程，一个轮胎坏了，之后她以 5 公里每小时的速度步行走完了剩余的路程，最终到她朋友家总共花了 44 分钟，Samia 步行了多少公里路程？结果保留一位小数。

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Problem 5

The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartile (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

一组数据$[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$的中位数是$Q_2 = 40$，第一个四分位数是$Q_1 = 33$，第三个四分位数是$Q_3 = 43$. 异常的数据定义为比第一个四分位数$(Q_1)$低超过 1. 5 倍的四分位距，或者比第三个四分位数(Q_3)高超过 1. 5 倍的四分位距，而这里的四分位距则定义为$Q_3 - Q_1$，那么这组数据有多少个异常的数?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 6

The circle having $(0,0)$ and $(8,6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

一个直径两端点分别为（0，0）和（8，6）的圆和 x 轴交于第二个点。问这第二个点的 x 坐标是多少？

(A) $4\sqrt{2}$ **(B)** 6 **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** $6\sqrt{2}$

Problem 7

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

函数$\sin(x)$和$\cos(x)$都是最小正周期是 2π的周期函数。那么函数$\cos(\sin(x))$的最小正周期是多少？

(A) $\frac{\pi}{2}$ **(B)** π **(C)** 2π **(D)** 4π **(E)** It's not periodic.

Problem 8

The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

某个矩形的短边和长边之比等于长边和对角线之比，那么这个矩形的短边和长边之比的平方是多少？

(A) $\frac{\sqrt{3}-1}{2}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{6}-1}{2}$

Problem 9

A circle has center $(-10,-4)$ and radius 13. Another circle has center $(3,9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x+y=c$. What is c?

一个圆的圆心为（-10，-4），半径是 13. 另一个圆的圆心为（3，9），半径是$\sqrt{65}$.通过这两个圆的两个交点的直线方程为$x+y=c$。那么 c 是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 3\sqrt{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Problem 10

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

在 Typico 高中，60％的学生喜欢跳舞，剩余的人不喜欢，在那些喜欢跳舞的人中，80％的人说他们喜欢跳舞，剩余的人说他们不喜欢跳舞，而在那些不喜欢跳舞的人中，90％的人说他们不喜欢跳舞，其余的人说他们喜欢跳舞，那些说自己不喜欢跳舞的学生当中，有多少比例实际上是喜欢跳舞的？

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ 33\frac{1}{3}\%$

Problem 11

Call a positive integer $monotonous$ if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

如果一个正整数是一位数字或者当从左往右读时，它的各个位上的数字形成一个严格递增或严格递减的数列，那么我们把此正整数称作单调的，例如，3，23578 和 987620 都是单调的，但是 88，7434 和 23557 不是。那么一共有多少个单调的正整数?

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

Problem 12

What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

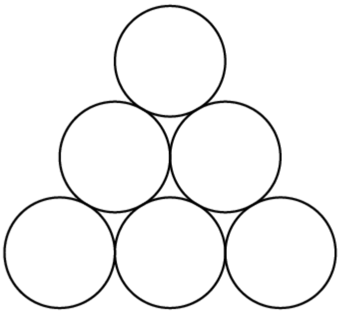
方程$z^{12} = 64$的所有根中，那些实部是正数的根之和是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ \sqrt{2}+2\sqrt{3} \qquad \textbf{(D)}\ 2\sqrt{2}+\sqrt{6} \qquad \textbf{(E)}\ (1+\sqrt{3})+(1+\sqrt{3})i$

Problem 13

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?

如下图所示，6 个圆盘中其中 3 个涂成蓝色，2 个涂成红色，1 个涂成绿色，如果两种涂色图案可以通过旋转或者对称重合，那么这 2 种涂色图案视作同一个。问一共有多少种不同的图案?



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Problem 14

An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

一种花饰冰激凌由成截头椎体形状的杯子和圆锥形冰激凌组成。杯子的高度为 4 英寸，下底直径 2 英寸，上底直径 4 英寸，圆锥形冰激凌高为 4 英寸，其底部恰好就是杯子的上底。求整个冰激凌的体积是多少立方英寸？

$\textbf{(A)}\ 8\pi \qquad \textbf{(B)}\ \frac{28\pi}{3} \qquad \textbf{(C)}\ 12\pi \qquad \textbf{(D)}\ 14\pi \qquad \textbf{(E)}\ \frac{44\pi}{3}$

Problem 15

Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3 \cdot CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

ABC 为一个等边三角形，延长边 *AB* 到 *B*′ 使得 *B*′ =3*AB*，类似的，延长边 *BC* 到 *C*′ 使得 *CC*′ =3*BC*，延长边 *CA* 到 *A*′ 使得 *AA*′ =3*CA*.则$\triangle A'B'C'$的面积和$\triangle ABC$的面积的比值是多少？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 37$

Problem 16

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

数字$21! = 51,090,942,171,709,440,000$的正整数因子的个数超过 60000 个，从这些因子中随机选择 1 个，这个数是奇数的概率是多少？

$\textbf{(A)}\ \frac{1}{21} \qquad \textbf{(B)}\ \frac{1}{19} \qquad \textbf{(C)}\ \frac{1}{18} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{11}{21}$

Problem 17

A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

抛一枚非标准硬币，正面朝上的概率是$\frac{2}{3}$，反面朝上的概率是$\frac{1}{3}$，每次实验的结果都是相互独立的。某个选手可以选择玩游戏 A 或者游戏 B，在游戏 A 中，她需要抛一枚硬币抛 3 次，若 3 次结果都一样，那她就赢了。在游戏 B 中，她需要抛一枚硬币抛 4 次，如果第一次和第二次的结果都一样，并且第三次和第四次的结果也一样，那么她就赢了。那么赢得游戏 A 的概率和赢得游戏 B 的概率相比，下列哪个说法是正确的？

（A）The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B. | 赢得游戏 A 的概率比赢得游戏 B 的概率少$\frac{4}{81}$

（B）The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B. | 赢得游戏 A 的概率比赢得游戏 B 的概率少$\frac{2}{81}$

（C）The probabilities are the same. | 概率是一样的.

（D）The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B. | 赢得游戏 A 的概率比赢得游戏 B 的概率多$\frac{2}{81}$

（E）The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B. | 赢得游戏 A 的概率比赢得游戏 B 的概率多$\frac{4}{81}$

Problem 18

The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

半径为 2 的圆的直径 *AB* 延长到位于圆外的点 *D*，使得 *BD*=3，选择一个点 *E*，使得 *ED*=5 且 *ED* 垂直于 *AD*，线段 *AE* 和圆的交点 *C* 位于 *A* 和 *E* 之间，那么$\triangle ABC$的面积是多少？

$\textbf{(A)}\ \frac{120}{37} \qquad \textbf{(B)}\ \frac{140}{39} \qquad \textbf{(C)}\ \frac{145}{39} \qquad \textbf{(D)}\ \frac{140}{37} \qquad \textbf{(E)}\ \frac{120}{31}$

Problem 19

Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

$N = 123456789101112\dots4344$是一个 79 位的数字，是由从 1 依次写到 44 的整数形成，问 *N* 除以 45 所得余数为多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 44$

Problem 20

Real numbers x and y are chosen independently and uniformly at random from the interval $(0,1)$.

What is the probability that $\lfloor\log_2 x\rfloor = \lfloor\log_2 y\rfloor$?

从区间$(0,1)$中独立且均匀的随机选择实数 *x* 和 *y*。那么 *x* 和 *y* 满足方程$\lfloor\log_2 x\rfloor = \lfloor\log_2 y\rfloor$的概率是多少？

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 21

Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

去年 Isabella 参加了 7 场数学考试并得到 7 个分数，每个分数都是 91 到 100 之间且包括 91 和 100 的整数。每场考试后，她都发现她考试的平均分是个整数。她第 7 场考试的分数是 95 分，那么她第 6 场考试得多少分?

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 94 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 98 \qquad \textbf{(E)}\ 100$

Problem 22

Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn---one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

Abby，Bernardo，Carl 和 Debra 玩一种游戏，一开始他们每个人都持有 4 枚硬币。这个游戏一共有 4 个回合。在每个回合的开始，罐子里放 4 个球：1 个绿色，1 个红色，2 个白色。每个选手不放回地随机从中抽取一个球，抽到绿球的选手就给抽到红球的选手一枚硬币。那么在第 4 个回合结束后，每个选手都有 4 枚硬币的概率是多少？

$\textbf{(A)}\ \frac{7}{576} \qquad \textbf{(B)}\ \frac{5}{192} \qquad \textbf{(C)}\ \frac{1}{36} \qquad \textbf{(D)}\ \frac{5}{144} \qquad \textbf{(E)}\ \frac{7}{48}$

Problem 23

The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$?

函数$y = f(x)$是个 3 次多项式，图像经过点$A(2,4)$，$B(3,9)$，$C(4,16)$。直线 AB，AC和 BC 又分别和$y = f(x)$的图像交于点 D，E和 F，并且点 D，E和 F 的 x-坐标之和是 24，那么$f(0)$是多少?

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ \frac{24}{5} \qquad \textbf{(E)}\ 8$

Problem 24

Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\frac{AB}{BC}$?

四边形 *ABCD* 的直角顶点是 *B* 和 *C*，$\triangle ABC \sim \triangle BCD$且 *AB*> *BC*. 四边形 *ABCD* 内部有一点 *E*，满足$\triangle ABC \sim \triangle CEB$，且$\triangle AED$的面积是$\triangle CEB$面积的 17 倍。问$\frac{AB}{BC}$是多少？

(A) $1+\sqrt{2}$ **(B)** $2+\sqrt{2}$ **(C)** $\sqrt{17}$ **(D)** $2+\sqrt{5}$ **(E)** $1+2\sqrt{3}$

Problem 25

A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \leq n \leq 2017$, can be the number of participants?

n 个人参加网上视频篮球循环赛。任何一个选手都可能是任意 5 人组成的一支篮球队的一员，但是不可能有两支队伍的 5 个选手是同样的 5 个人。网上的统计数据显示了一个令人惊奇的事实：在所有可能的从 *n* 个选手中选择的 9 人小组中，平均下来每个这样的 9 人小组所包含的完整一支 5 人球队的数目，和在所有可能的从 *n* 个选手中选择的 8 人小组中，平均下来每个这样的 8 人小组所包含的完整一支 5 人球队的数目，这两个手均数互为倒数。当$9 \leq n \leq 2017$时，有多少个这样的 *n* 值，可以是选手的个数？

(A) 477 **(B)** 482 **(C)** 487 **(D)** 557 **(E)** 562

2017 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
E	E	D	C	B	D	B	C	A	D	B	D	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	E	B	D	D	C	D	E	B	D	D	D	