2012 AMC12B

Problem 1

Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

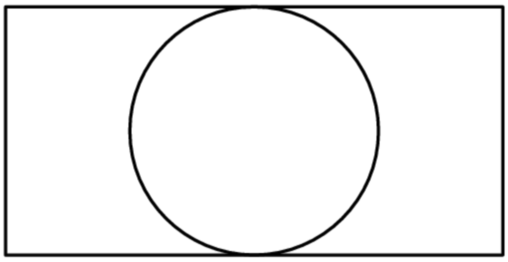
Pearl Creek 小学的每一个三年级教室都有 18 个学生和 2 只宠物兔，问四个三年级教室的学生总数比宠物兔总数多出多少个?

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 80$

Problem 2

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?

如图所示，一个半径为 5 的圆和一个矩形内切，矩形的长和宽之比为 2:1，问矩形的面积是多少?



$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 200$

Problem 3

For a science project, Sammy observed a chipmunk and squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

为了完成一项科学作业，Sammy 观察花栗鼠和松鼠在洞里藏橡果的情况。花栗鼠在它所挖的每个洞里藏 3 颗橡果，而松鼠在它所挖的每个洞里藏 4 颗橡果，它们藏的橡果的总数一样多，但是松鼠所需要的洞比花栗鼠的少 4 个，问花栗鼠藏了多少颗橡果?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 54$

Problem 4

Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater that the value of Diana's money?

假设 1 欧元可以兑换 1.3 美元，若 Diana 有 500 美元，Eticnne 有 400 欧元，那么 Etienne 的钱比 Diana 的钱多百分之多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6.5 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 13$

Problem 5

Two integers have a sum of 26. When two more integers are added to the first two, the sum is 41. Finally, when two more integers are added to the sum of the previous 4 integers, the sum is 57. What is the minimum number of even integers among the 6 integers?

两个整数的和为 26，当在这两个整数的和的基础上，再加上另外两个整数，总和是 41，当在这前 4 个整数之和的基础上，再加上两个整数，和变成 57，问这 6 个整数中，偶数的个数最少是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 6

In order to estimate the value of $x-y$ where x and y are real numbers with $x>y>0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

x 和 y 都是实数且 x>y>0，为了估计 x-y 的值，Xiaoli 通过把 x 增加一定量使之五入到一个整数，通过把 y 减小同样的量使之四舍到一个整数，然后分别把 x 和 y 四舍五入后的值相减，问下面哪句话是对的？

$\textbf{(A)}$ Her estimate is larger than $x-y$ | 她的估算比$x-y$大

$\textbf{(B)}$ Her estimate is smaller than $x-y$ | 她的估算比$x-y$小

$\textbf{(C)}$ Her estimate equals $x-y$ | 她的估算等于$x-y$

$\textbf{(D)}$ Her estimate equals $y-x$ | 她的估算等于$y-x$

$\textbf{(E)}$ Her estimate is 0 | 她的估算等于 0

Problem 7

Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

有一些挂在一根绳子上的小灯，它们相互间隔 6 英寸，依照这样的顺序：红，红，绿，绿，绿，红，红，绿，绿，绿，等等，按照这种 2 个红灯后面跟着 3 个绿灯的方式继续。那么第 3 个红灯和第 21 个红灯之间的距离是多少英尺？

注意:1 英尺等于 12 英寸。

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 18.5 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 20.5 \qquad \textbf{(E)}\ 22.5$

Problem 8

A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

一个甜点厨师给一周的每一天都准备了甜点，从周日开始，每天的甜点是蛋糕、馅饼、冰淇淋或者布丁，不能连续两天都准备同一种甜点，同时因为周五有生日，所以周五必须是蛋糕，那么一周一共有多少种不同的甜点菜单？

$\textbf{(A)}\ 729 \qquad \textbf{(B)}\ 972 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 2187 \qquad \textbf{(E)}\ 2304$

Problem 9

It takes Clea 60 seconds to walk down an escalator when it is not moving, and 24 seconds when it is moving. How many seconds would it take Clea to ride the escalator down when she is not walking?

当电梯不运行时，Clea 从电梯走下来需要 60 秒，但是当电梯运行时，他从电梯走下来只要 24 秒，问当电梯运行时，如果 Clea 站在电梯上不动，从电梯下来需要多少秒？

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 52$

Problem 10

What is the area of the polygon whose vertices are the points of intersection of the curves $x^2+y^2=25$ and $(x-4)^2+9y^2=81$?

以曲线$x^2+y^2=25$和曲线$(x-4)^2+9y^2=81$的交点为顶点的多边形的面积是多少？

$\textbf{(A)}\ 24\qquad\textbf{(B)}\ 27\qquad\textbf{(C)}\ 36\qquad\textbf{(D)}\ 37.5\qquad\textbf{(E)}\ 42$

Problem 11

In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:

$$132_A+43_B=69_{A+B}.$$

What is $A+B$?

在下面的方程中，A 和 B 是连续正整数，且 A，B 和 $A+B$ 代表进制：

$$132_A+43_B=69_{A+B}.$$

问 $A+B$ 等于多少？

$\textbf{(A)}\ 9\qquad\textbf{(B)}\ 11\qquad\textbf{(C)}\ 13\qquad\textbf{(D)}\ 15\qquad\textbf{(E)}\ 17$

Problem 12

How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

有多少个由 0 和 1 组成的长度为 20 的序列满足：所有的 0 都连续，或者所有的 1 都连续，或者两者兼具？

$\textbf{(A)}\ 190\qquad\textbf{(B)}\ 192\qquad\textbf{(C)}\ 211\qquad\textbf{(D)}\ 380\qquad\textbf{(E)}\ 382$

Problem 13

Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers, each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas will have at least one point in common?

两个抛物线方程为$y = x^2 + ax + b$和$y = x^2 + cx + d$，其中 a，b，c，d 都是整数，且每个的取值都来自于掷一个标准六面骰子得到的数，那么这两个抛物线至少有一个公共点的概率是多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{25}{36} \qquad \textbf{(C)}\ \frac{5}{6} \qquad \textbf{(D)}\ \frac{31}{36} \qquad \textbf{(E)}\ 1$

Problem 14

Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she addes 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

Bernardo 和 Silvia 玩有如下规则的游戏：从 0-999（包含 0 和 999）中选择一个整数，把它给 Bernardo，只要 Bernardo 拿到一个数，他就会把它乘以 2，然后把结果给 Silvia，只要 Silvia 得到一个数字，她就会把这个数字加 50，然后把结果再给 Bernardo。赢得比赛的人是最后那个产生一个小于 1000 的数的人，令 N 表示可以使得 Bernardo 赢的最小的初始值，那么 N 的各个位上数字之和为多少？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Problem 15

Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

Jesse 沿着半径为 12 的一个圆形纸盘的两条半径剪出了两个扇形，小的扇形的圆心角是 120 度，他用这两个扇形作为圆锥侧面做了两个圆锥，问小圆锥的体积和大圆锥的体积之比是多少?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{\sqrt{10}}{10} \qquad \textbf{(D)}\ \frac{\sqrt{5}}{6} \qquad \textbf{(E)}\ \frac{\sqrt{5}}{5}$

Problem 16

Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those girls but disliked by the third. In how many different ways is this possible?

Amy，Beth 和 Jo 听四首不同的歌曲，讨论她们喜欢哪几首，不存在她们三个人都喜欢的歌。并且，对于这三对女孩中的每一对，至少有一首歌被这一对女孩都喜欢，但不会被第三个女孩喜欢，问她们一共有多少种可能的方式喜欢歌曲？

$\textbf{(A)}\ 108 \qquad \textbf{(B)}\ 132 \qquad \textbf{(C)}\ 671 \qquad \textbf{(D)}\ 846 \qquad \textbf{(E)}\ 1105$

Problem 17

Square $PQRS$ lies in the first quadrant. Points $(3,0), (5,0), (7,0),$ and $(13,0)$ lie on lines $SP, RQ, PQ,$ and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

正方形 *PQRS* 位于第一象限，点（3，0），（5，0），（7，0）和（13，0）分别在直线 *SP*，*RQ*，*PQ* 和 *SR* 上，那么正方形 *PQRS* 的中心的横坐标和纵坐标之和是多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 6.2 \qquad \textbf{(C)}\ 6.4 \qquad \textbf{(D)}\ 6.6 \qquad \textbf{(E)}\ 6.8$

Problem 18

Let $(a_1,a_2,\dots,a_{10})$ be a list of the first 10 positive integers such that for each $2\le i\le10$ either a_i+1 or a_i-1 or both appear somewhere before a_i in the list. How many such lists are there?

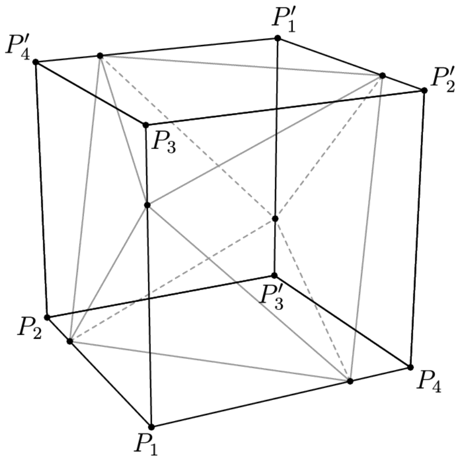
令（a1，a2，…，a10）是前 10 个正整数所组成的一列数字，满足对于每个$2\le i\le10$，*ai+1* 或者 *ai-1* 或者两者都排在 *ai* 的前面，问一共有多少种这样的一列数？

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 512 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 181,440 \qquad \textbf{(E)}\ 362,880$

Problem 19

A unit cube has vertices $P_1, P_2, P_3, P_4, P_1', P_2', P_3',$ and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \le i \le 4,$ vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments P_1P_2, P_1P_3, P_1P_4, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$. What is the octahedron's side length?

一个单位正方体顶点为$P_1, P_2, P_3, P_4, P_1', P_2', P_3',P_4'$，顶点$P_2$，$P_3$，$P_4$ 都与P_1相邻，并且对于$1 \le i \le 4,$顶点P_i和P_i'位置相对。已知线段P_1P_2，P_1P_3，P_1P_4，$P_1'P_2'$，$P_1'P_3'$，$P_1'P_4'$上都分别有某个正八边形的一个顶点，问这个正八边形的边长是多少？



$\textbf{(A)}\ \frac{3\sqrt{2}}{4} \qquad \textbf{(B)}\ \frac{7\sqrt{6}}{16} \qquad \textbf{(C)}\ \frac{\sqrt{5}}{2} \qquad \textbf{(D)}\ \frac{2\sqrt{3}}{3} \qquad \textbf{(E)}\ \frac{\sqrt{6}}{2}$

Problem 20

A trapezoid has side lengths 3, 5, 7, and 11. The sums of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of any prime. What is the greatest integer less than or equal to $r_1+r_2+r_3+n_1+n_2$?

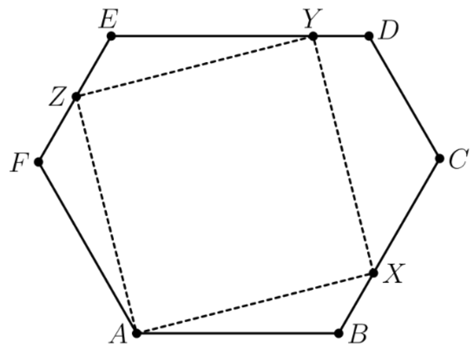
一个梯形的边长是 3, 5, 7 和 11，满足这个条件的所有可能的梯形的面积之和可以写成$r_1\sqrt{n_1}+r_2\sqrt{n_2}+r_3$，这里$r_1$，$r_2$和$r_3$都是有理数，且$n_1$和$n_2$是不能被任何质数的平方所整除的正整数。那么小于或等于$r_1+r_2+r_3+n_1+n_2$最大整数是多少？

$\textbf{(A)}\ 57 \qquad \textbf{(B)}\ 59 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 63 \qquad \textbf{(E)}\ 65$

Problem 21

Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$, and $EF = 41(\sqrt{3}-1)$. What is the side-length of the square?

正方形 *AXYZ* 内接在等角六边形 *ABCDEF* 中，点 *X* 在线段 *BC* 上，点 *Y* 在线段 *DE* 上，点 *Z* 在线段 *EF* 上。假设$AB = 40$，$EF = 41(\sqrt{3}-1)$，那么这个正方形的边长是多少？



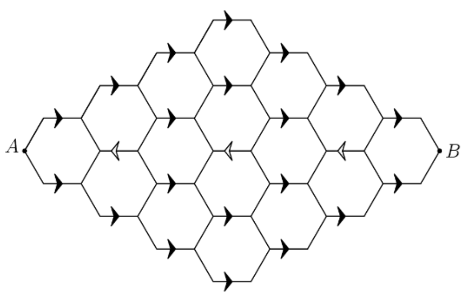
(A) $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2}+\frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3}+16$

(D) $20\sqrt{2}+13\sqrt{3}$ **(E)** $21\sqrt{6}$

Problem 22

A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?

一只虫子沿着如下图所示的六边形方格的边从 A 爬到 B，在标有箭头的线段爬行时，只能沿着箭头的方向爬行，且虫子不能在同一根线段上爬行超过 1 次，问一共有多少种不同的路径?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

Problem 23

Consider all polynomials of a complex variable, $P(z)=4z^4+az^3+bz^2+cz+d$, where $a,b,c,$ and d are integers, $0\le d\le c\le b\le a\le 4$, and the polynomial has a zero z_0 with $|z_0|=1.$ What is the sum of all values $P(1)$ over all the polynomials with these properties?

考虑未知数为复数的所有多项式$P(z)=4z^4+az^3+bz^2+cz+d$，其中 a，b，c，d 都是整数，$0\le d\le c\le b\le a\le 4$，并且多项式有一个零点$z_0$，且$|z_0|=1$.问满足这些性质的所有多项式的$P(1)$的值之和是多少？

$\textbf{(A)}\ 84 \qquad \textbf{(B)}\ 92 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 108 \qquad \textbf{(E)}\ 120$

Problem 24

Define the function f_1 on the positive integers by setting $f_1(1)=1$ and if $n=p_1^{e_1}p_2^{e_2}\cdots p_k^{e_k}$ is the prime factorization of $n>1$, then $f_1(n)=(p_1+1)^{e_1-1}(p_2+1)^{e_2-1}\cdots (p_k+1)^{e_k-1}$. For every $m\ge 2$, let $f_m(n)=f_1(f_{m-1}(n))$. For how many Ns in the range $1\le N\le 400$ is the sequence $(f_1(N),f_2(N),f_3(N),\dots)$ unbounded?

Note: A sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

f_1为一个定义在正整数上的函数，且$f_1(1)=1$. 若$n=p_1^{e_1}p_2^{e_2}\cdots p_k^{e_k}$是$n>1$的质因数分解，那么$f_1(n)=(p_1+1)^{e_1-1}(p_2+1)^{e_2-1}\cdots (p_k+1)^{e_k-1}$.对于每一个$m\ge 2$，令$f_m(n)=f_1(f_{m-1}(n))$. 对于$1\le N\le 400$，存在多少个$N$，使得数列$(f_1(N),f_2(N),f_3(N),\dots)$无上限？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 19$

Problem 25

Let $S=\{(x,y): x\in\{0,1,2,3,4\}, y\in\{0,1,2,3,4,5\},\ \text{and}\ (x,y)\neq(0,0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t=\triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t)=\tan(\angle CBA)$. What is

$$\prod_{t\in T} f(t)?$$

令集合$S=\{(x,y): x\in\{0,1,2,3,4\}, y\in\{0,1,2,3,4,5\},\ \text{and}\ (x,y)\neq(0,0)\}$. T是三个顶点都在 S 中的直角三角形组成的集合。对于每一个直角三角形$t=\triangle ABC$，其中顶点 A，B和 C的顺序为逆时针，且 A 为直角顶点，令$f(t)=\tan(\angle CBA)$，则 $\prod_{t\in T} f(t)$ 是多少？

(A) 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$

2012 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	D	B	A	A	E	A	B	B	C	D/E	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
A	C	B	C	B	A	D	A	E	B	D	B	

* 第 12 题：D or E (both were accepted)