2010 AMC12B

Problem 1

Makarla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

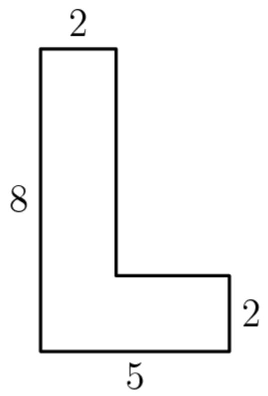
Markala 在她工作的 9 小时里参加了 2 次会议。第一场会议耗时 45 分钟，第二场会议时长是第一场的 2 倍。问她工作总时间的百分之多少是花在了会议上？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 35$

Problem 2

A big L is formed as shown. What is its area?

一个大的L字形如下图所示。问它的面积是多少？



$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 28 \qquad \textbf{(E)}\ 30$

Problem 3

A ticket to a school play cost x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of $\$48$, and a group of 10th graders buys tickets costing a total of $\$64$. How many values for x are possible?

一张学校戏剧票要花x美元，其中x是整数。一组九年级学生买的票总共花了 48 美元，一组十年级学生买的票总共花了 64 美元。问 x 有多少个可能的值？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 4

A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

某个月共有 31 天，其中周一的天数和周三的天数相同。一周 7 天有多少天可能成为这个月的第一天？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 5

Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?

幸运的 Lany 的老师叫他分别将值代入a, b, c, d和e，以求得表达式$a-(b-(c-(d+e)))$的值。Larry 忽略了括号，但加减运算是正确的，结果碰巧得到了正确答案。已知 Larry 分别用 1，2，3，4 替代a, b, c和 d，那么 Larry 用什么数替代了e?

$\textbf{(A)}\ -5 \qquad \textbf{(B)}\ -3 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 5$

Problem 6

At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

在某学年的开始，Well 先生数学课上的 50％的学生对于“你喜欢数学吗”这个问题的回答为“是”，剩下 50％的学生回答为“否”。在这学年结束时，70％的学生回答为“是”，剩下 30％的学生回答为“否”。总的来说，$x\%$的学生在学年开始和学年结束时给出了不同的回答。那么x的最大可能值和最小可能值之差是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 80$

Problem 7

Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

Shelby 在不下雨的情况下以每小时 30 英里的速度驾驶她的滑板车，在下雨的情况下以每小时 20 英里的速度驾驶。今天，她早上驾驶时天气晴朗，晚上驾驶时天下着雨，结果总共用了 40 分钟驾驶了 16 英里。问她在雨中驾驶了多少分钟？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 30$

Problem 8

Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

欧几里得市的每一所高中都会派出一支由 3 名学生组成的队伍去参加某个数学竞赛。每个参赛选手都得到了不同的分数。Andrea 的分数在所有这些学生中是个中位数，并且她的分数是她所在队的最高分。Andrea 的队友 Beth 和 Carla 分别排在第 37 名和第 64 名．问这个城市总共有多少所学校?

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 23 \qquad \textbf{(C)}\ 24 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 26$

Problem 9

Let n be the smallest positive integer such that n is divisible by 20, n^2 is a perfect cube, and n^3 is a perfect square. What is the number of digits of n?

令n为最小的正整数，满足n能被 20 整除，n^2是个完全立方数，且n^3也是个完全平方数。问n是个几位数？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 10

The average of the numbers $1, 2, 3, \cdots, 98, 99,$ and x is $100x$. What is x?

数字$1, 2, 3, \cdots, 98, 99$,x的平均值为$100x$，问x的值是多少？

(A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

Problem 11

A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

从所有位于 1000 到 10，000 之间的回环数中随机抽取 1 个，问这个回环数能被 7 整除的概率是多少？

(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

Problem 12

For what value of x does

$\log_{\sqrt{2}}\sqrt{x}+\log_2 x+\log_4 x^2+\log_8 x^3+\log_{16} x^4=40?$

当x为何值时，满足式子$\log_{\sqrt{2}}\sqrt{x}+\log_2 x+\log_4 x^2+\log_8 x^3+\log_{16} x^4=40?$

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256 **(E)** 1024

Problem 13

In $\triangle ABC$, $\cos(2A-B)+\sin(A+B)=2$ and $AB=4$. What is BC?

在$\triangle ABC$中，$\cos(2A-B)+\sin(A+B)=2$，且$AB=4$，问BC是多少？

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $2\sqrt{3}$

Problem 14

Let a, b, c, d, and e be positive integers with $a+b+c+d+e=2010$ and let M be the largest of the sums $a+b$, $b+c$, $c+d$ and $d+e$. What is the smallest possible value of M?

a，b，c，d，e 均为正整数，满足 a+b+c+d+e=2010。令M为 a+b，b+C，c+d 和 d+e 这些和的最大值。问M的最小可能值是多少？

(A) 670 **(B)** 671 **(C)** 802 **(D)** 803 **(E)** 804

Problem 15

For how many ordered triples (x,y,z) of nonnegative integers less than 20 are there exactly two distinct elements in the set $\{i^x,(1+i)^y,z\}$, where $i=\sqrt{-1}$?

有多少个有序三元组(x,y,z),其中 x,y,z 均为小于 20 的非负整数,满足集合$\{i^x,(1+i)^y,z\}$恰有 2 个不同的元素？这里$i=\sqrt{-1}$。

(A) 149 **(B)** 205 **(C)** 215 **(D)** 225 **(E)** 235

Problem 16

Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1,2,3,\dots,2010\}$. What is the probability that $abc+ab+a$ is divisible by 3?

正整数a，b和c独立、随机且有放回地从集合｛1，2，3，．..，2010｝中抽取。问$abc+ab+a$能被 3 整除的概率是多少？

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

Problem 17

The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

一个3×3的阵列包含了 1 到 9 的所有数字，满足每行每列的数字都以升序排列。一共有多少种这样的阵列？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 60$

Problem 18

A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

一只青蛙跳了 3 次，每次跳跃的长度均为 1 米，跳跃的方向是随机且独立选择的。问青蛙最后的位置距离它的起点不超过 1 米的概率是多少？

$\textbf{(A)}\ \dfrac{1}{6} \qquad \textbf{(B)}\ \dfrac{1}{5} \qquad \textbf{(C)}\ \dfrac{1}{4} \qquad \textbf{(D)}\ \dfrac{1}{3} \qquad \textbf{(E)}\ \dfrac{1}{2}$

Problem 19

A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

Raiders 队和 Wildcats 队之间的一场高中篮球赛在第一节打成平局。已知 Raiders 队的 4 节比赛所得的 4 个分数成递增等比数列，而 Wildcats 队的 4 节比赛所得的 4 个分数成递增的等差数列。在第 4 节结束后，Raiders 队的总分比 Wildcats 的总分高 1 分，2 个队伍的总分均不超过 100 分。问这 2 支队伍在前 2 节比赛里的总得分是多少分？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 33 \qquad \textbf{(E)}\ 34$

Problem 20

A geometric sequence (a_n) has $a_1 = \sin x$, $a_2 = \cos x$, and $a_3 = \tan x$ for some real number x. For what value of n does $a_n = 1 + \cos x$?

一个等比数列(a_n)满足$a_1 = \sin x$，$a_2 = \cos x$，$a_3 = \tan x$，这里的x是一个实数。问n为何值时，满足$a_n = 1 + \cos x$？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 21

Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a\text{, and}$$

$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

令$a > 0$，且多项式$P(x)$的各项系数均为整敏，满足

$$P(1) = P(3) = P(5) = P(7) = a$$

$$P(2) = P(4) = P(6) = P(8) = -a$$

那么a的最小可能值是多少？

$\textbf{(A)}\ 105 \qquad \textbf{(B)}\ 315 \qquad \textbf{(C)}\ 945 \qquad \textbf{(D)}\ 7! \qquad \textbf{(E)}\ 8!$

Problem 22

Let $ABCD$ be a cyclic quadrilateral. The side lengths of $ABCD$ are distinct integers less than 15 such that $BC \cdot CD = AB \cdot DA$. What is the largest possible value of BD?

$ABCD$为圆内接四边形，$ABCD$的边长为小于 15 的不同的整数，满足$BC \cdot CD = AB \cdot DA$，那么$BD$的最大可能值是多少？

$\textbf{(A)}\ \sqrt{\dfrac{325}{2}} \qquad \textbf{(B)}\ \sqrt{185} \qquad \textbf{(C)}\ \sqrt{\dfrac{389}{2}} \qquad \textbf{(D)}\ \sqrt{\dfrac{425}{2}} \qquad \textbf{(E)}\ \sqrt{\dfrac{533}{2}}$

Problem 23

Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeros at $x=-23,-21,-17,$ and -15, and $Q(P(x))$ has zeros at $x=-59,-57,-51$ and -49. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

首项系数为 1 的二次多项式$P(x)$和$Q(x)$，有这样的性质：$P(Q(x))$的零点为 x=-23，-21，-17 和-15，$Q(P(x))$的零点为 x=-59，-57，-51 和-49。那么$P(x)$和$Q(x)$的最小值之和是多少？

$\textbf{(A)}\ -100 \qquad \textbf{(B)}\ -82 \qquad \textbf{(C)}\ -73 \qquad \textbf{(D)}\ -64 \qquad \textbf{(E)}\ 0$

Problem 24

The set of real numbers x for which

$$\frac{1}{x-2009}+\frac{1}{x-2010}+\frac{1}{x-2011}\ge 1$$

is the union of intervals of the form $a<x\le b$. What is the sum of the lengths of these intervals?

满足不等式

$$\frac{1}{x-2009}+\frac{1}{x-2010}+\frac{1}{x-2011}\ge 1$$

的所有实数x组成的集合是$a<x\le b$的若干区间的并集。问这些区间的长度之和是多少？

$\textbf{(A)}\ \frac{1003}{335} \qquad \textbf{(B)}\ \frac{1004}{335} \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ \frac{403}{134} \qquad \textbf{(E)}\ \frac{202}{67}$

Problem 25

For every integer $n \geq 2$, let $\operatorname{pow}(n)$ be the largest power of the largest prime that divides n. For example $\operatorname{pow}(144) = \operatorname{pow}(2^4 \cdot 3^2) = 3^2$. What is the largest integer m such that 2010^m divides

$$\prod_{n=2}^{5300} \operatorname{pow}(n)$$
?

对于每个整数$n \geq 2$，令$\operatorname{pow}(n)$表示能够整除n的最大质因数的最大幂。例如

$\operatorname{pow}(144) = \operatorname{pow}(2^4 \cdot 3^2) = 3^2$。问使得$2010^m$能够整除

$$\prod_{n=2}^{5300} \operatorname{pow}(n)$$

的最大整数m是多少？

$\textbf{(A)}\ 74 \qquad \textbf{(B)}\ 75 \qquad \textbf{(C)}\ 76 \qquad \textbf{(D)}\ 77 \qquad \textbf{(E)}\ 78$

2010 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	A	E	B	D	D	C	B	E	B	E	D	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	D	E	D	C	E	E	B	D	A	C	D	