2010 AMC12A

Problem 1

What is $(20-(2010-201))+(2010-(201-20))$?

表达式$(20-(2010-201))+(2010-(201-20))$的值是多少？

$\textbf{(A)}\ -4020 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 401 \qquad \textbf{(E)}\ 4020$

Problem 2

A ferry boat shuttles tourists to an island every hour starting at 10 AM until its last trip, which starts at 3 PM. One day the boat captain notes that on the 10 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

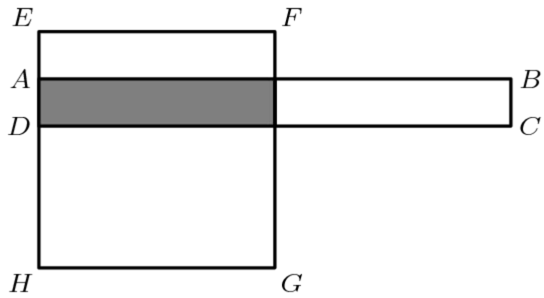
一艘渡轮从上午 10 点开始，每隔一小时就会把游客送到一个岛上，直到最后一班（最后一班下午 3 点出发）。一天，船长注意到，上午 10 点的这一班，渡轮上有 100 名游客，而且在后续的每一班，游客人数都比前一班少 1 人。问那一天渡轮总共运送了多少游客到岛上？

$\textbf{(A)}\ 585 \qquad \textbf{(B)}\ 594 \qquad \textbf{(C)}\ 672 \qquad \textbf{(D)}\ 679 \qquad \textbf{(E)}\ 694$

Problem 3

Rectangle $ABCD$, pictured below, shares 50% of its area with square $EFGH$. Square $EFGH$ shares 20% of its area with rectangle $ABCD$. What is $\frac{AB}{AD}$?

矩形$ABCD$如下图所示，和正方形$EFGH$重叠部分的面积占了它总面积的 50%．而重叠部分的面积占了正方形$EFGH$总面积的 20%．问$\frac{AB}{AD}$是多少？



(A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

Problem 4

If $x < 0$, then which of the following must be positive?

若$x < 0$，那么下面哪个一定是正数？

(A) $\frac{x}{|x|}$ **(B)** $-x^2$ **(C)** -2^x **(D)** $-x^{-1}$ **(E)** $\sqrt[3]{x}$

Problem 5

Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot a bullseye scores 10 points, with other possible scores being 8, 4, 2, and 0 points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bullseyes she will be guaranteed victory. What is the minimum value for n?

在 100 杆射箭比赛进行到一半时，Chelsea 领先 50 分。每一次射击，射中靶心得 10 分，其他可能得分分别为 8 分、4 分、2 分和 0 分。Chelsca 每次射击都至少得 4 分。如果 Chelsea 接下来的n次射击都射中靶心，那么她一定会获胜。问n的最小值是多少？

(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

Problem 6

A $palindrome$, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x+32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

回环数是指具有这样的特点的数：当将它的各位上数字从右向左重新排列后，形成的数和原来的数一样。例如 83438 就是个回环数。已知：x和$x+32$ 分别是 3 位和 4 位回环数。那么x的各个位上数字之和是多少？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 24$

Problem 7

Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

Logan 把他的城镇按比例做了个模型。城镇的水塔高 40 米，顶部是一个能装 100，000 升水的球体。Logan 的模型水塔能装 0.1 升水，问 Logan 应该把他的模型塔的高做多少米？

$\textbf{(A)}\ 0.04 \qquad \textbf{(B)}\ \frac{0.4}{\pi} \qquad \textbf{(C)}\ 0.4 \qquad \textbf{(D)}\ \frac{4}{\pi} \qquad \textbf{(E)}\ 4$

Problem 8

Triangle ABC has $AB=2\cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

三角形ABC中，$AB=2\cdot AC$。点D和E分别在$\overline{AB}$和$\overline{BC}$上，满足$\angle BAE = \angle ACD$。令F为线段AE和CD的交点，假设$\triangle CFE$为等边三角形，那么$\angle ACB$是多少度？

$\textbf{(A)}\ 60^\circ \qquad \textbf{(B)}\ 75^\circ \qquad \textbf{(C)}\ 90^\circ \qquad \textbf{(D)}\ 105^\circ \qquad \textbf{(E)}\ 120^\circ$

Problem 9

A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

一个正方体的棱长为 3 英寸，每个面上都切出一个 2 英寸×2 英寸的正方形洞。每一个切口的边都平行于正方体的边，并且每个洞都贯穿整个正方体。问剩余立体图形的体积是多少立方英寸？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Problem 10

The first four terms of an arithmetic sequence are p, 9, $3p-q$, and $3p+q$. What is the 2010^th term of this sequence?

一个等差数列的前 4 项为p，9，$3p-q$和$3p+q$，那么这个数列的第 2010 项是多少？

$\textbf{(A)}\ 8041 \qquad \textbf{(B)}\ 8043 \qquad \textbf{(C)}\ 8045 \qquad \textbf{(D)}\ 8047 \qquad \textbf{(E)}\ 8049$

Problem 11

The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b?

方程$7^{x+7} = 8^x$的解可以写成$x = \log_b 7^7$这种形式，问b是多少？

$\textbf{(A)}\ \frac{7}{15} \qquad \textbf{(B)}\ \frac{7}{8} \qquad \textbf{(C)}\ \frac{8}{7} \qquad \textbf{(D)}\ \frac{15}{8} \qquad \textbf{(E)}\ \frac{15}{7}$

Problem 12

In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

在一片神奇的沼泽中，有 2 种会说话的两栖动物：一种是癞蛤蟆，它们总是说真话，还有一种是青蛙，它们总是说谎话。现在有 4 只两栖动物，它们分别是 Brian，Chris，LeRoy 和 Mike，都住在这片沼泽中，并有如下对话：

Brian 说：“Mike 和我是不同的种类。”

Chris 说：“LeRoy 是一只青蛙。”

LeRoy 说：“Chris 是一只青蛙。”

Mike 说：“我们这 4 只动物中，至少有 2 只是癞蛤蟆。”

问这 4 只两栖动物中有几只是青蛙？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 13

For how many integer values of k do the graphs of $x^2+y^2=k^2$ and $xy=k$ not intersect?

整数k可以取多少种值，使得$x^2+y^2=k^2$和$xy=k$两者的图像不相交？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 8$

Problem 14

Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

非退化$\triangle ABC$的边长均为整数，$\overline{BD}$是条角平分线， $AD = 3$，$DC = 8$。问它的周长的最小可能值是多少？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 33 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 37$

Problem 15

A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

一种被改造后的硬币扔一次正面朝上的概率小于$\frac{1}{2}$，当这枚硬币被扔了 4 次后，正面朝上和反面朝上次数相等的概率为$\frac{1}{6}$。问硬币扔一次正面朝上的概率是多少？

$\textbf{(A)}\ \frac{\sqrt{15}-3}{6} \qquad \textbf{(B)}\ \frac{6-\sqrt{6\sqrt{6}+2}}{12} \qquad \textbf{(C)}\ \frac{\sqrt{2}-1}{2} \qquad \textbf{(D)}\ \frac{3-\sqrt{3}}{6} \qquad \textbf{(E)}\ \frac{\sqrt{3}-1}{2}$

Problem 16

Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,...,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,...,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

Bermardo 从集合｛1，2，3，4，5，6，7，8，9｝中随机选择 3 个不同的数，并把它们降序排列形成一个 3 位数。Silvia 从集合｛1，2，3，4，5，6，7，8｝中随机选择 3 个不同的数，也把它们降序排列形成一个 3 位数。问 Bernardo 的数比 Silvia 的数大的概率是多少？

$\textbf{(A)}\ \frac{47}{72} \qquad \textbf{(B)}\ \frac{37}{56} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{49}{72} \qquad \textbf{(E)}\ \frac{39}{56}$

Problem 17

Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

等角六边形$ABCDEF$的边长为$AB = CD = EF = 1$，$BC = DE = FA = r$，已知$\triangle ACE$的面积是六边形面积的 70%。那么r的所有可能值之和是多少？

$\textbf{(A)}\ \frac{4\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{10}{3} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ \frac{17}{4} \qquad \textbf{(E)}\ 6$

Problem 18

A 16-step path is to go from $(-4,-4)$ to $(4,4)$ with each step increasing either the x-coordinate or the y-coordinate by 1. How many such paths stay outside or on the boundary of the square $-2 \le x \le 2$, $-2 \le y \le 2$ at each step?

一条从（−4，−4）到（4，4）的 16 步路径的每一步要么是x坐标增加 1，要么是y坐标增加 1。有多少条这样的路径，满足每一步都在正方形$-2 \le x \le 2$，$-2 \le y \le 2$之外或者位于其边缘上？

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 144 \qquad \textbf{(C)}\ 1568 \qquad \textbf{(D)}\ 1698 \qquad \textbf{(E)}\ 12,800$

Problem 19

Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the $k\text{th}$ position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

一条直线上有 2010 个盒子，每个盒子内都装有一颗红色玻璃球，并且对于$1 \le k \le 2010$，在第κ个位置的盒子也包含κ颗白色玻璃球。Isabella 从第一个盒子开始，连续地依次从每个盒子里随机抽取一颗玻璃球。当她第一次抽到红色玻璃球后，她便停止。令 $P(n)$ 表示当 Isabella 抽了恰好n颗玻璃球后停止的概率。问使得 $P(n) < \frac{1}{2010}$ 的最小的 n 是多少？

$\textbf{(A)}\ 45 \qquad \textbf{(B)}\ 63 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 201 \qquad \textbf{(E)}\ 1005$

Problem 20

Arithmetic sequences (a_n) and (b_n) have integer terms with $a_1 = b_1 = 1 < a_2 \le b_2$ and $a_n b_n = 2010$ for some n. What is the largest possible value of n?

等差数列(a_n)和(b_n)的项均为整数，满足$a_1 = b_1 = 1 < a_2 \le b_2$，且对于某个$n$，有$a_n b_n = 2010$。问$n$的最大可能值是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 288 \qquad \textbf{(E)}\ 2009$

Problem 21

The graph of $y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$ lies above the line $y = bx + c$ except at three values of x, where the graph and the line intersect. What is the largest of these values?

$y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$的图像除了和直线$y = bx + c$相交于$x$的 3 个值外，其余部分均在这条直线上方。问这 3 个值中最大的值是多少？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 8$

Problem 22

What is the minimum value of $|x-1| + |2x-1| + |3x-1| + \cdots + |119x - 1|$?

$|x-1| + |2x-1| + |3x-1| + \cdots + |119x - 1|$的最小值是多少？

$\textbf{(A)}\ 49 \qquad \textbf{(B)}\ 50 \qquad \textbf{(C)}\ 51 \qquad \textbf{(D)}\ 52 \qquad \textbf{(E)}\ 53$

Problem 23

The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

$90!$的最后两个非零数字组成的数是n，问n是多少？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 52 \qquad \textbf{(E)}\ 68$

Problem 24

Let $f(x) = \log_{10}\left(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x)\right)$. The intersection of the domain of $f(x)$ with the interval $[0, 1]$ is a union of n disjoint open intervals. What is n?

令$f(x) = \log_{10}\left(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x)\right)$，则$f(x)$的定义域和区间的交集是$n$个不连续的开区间的并集。求$n$是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 22 \qquad \textbf{(E)}\ 36$

Problem 25

Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

如果一个四边形通过旋转或平移得到另一个四边形，则这两个四边形被认为是同一个。问边长为整数，周长等于 32 的不同的圆内接凸四边形有多少个？

$\textbf{(A)}\ 560 \qquad \textbf{(B)}\ 564 \qquad \textbf{(C)}\ 568 \qquad \textbf{(D)}\ 1498 \qquad \textbf{(E)}\ 2255$

2010 AMC 12A Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C	A	E	D	C	E	C	C	A	A	C	D	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	D	B	E	D	A	C	A	A	A	B	C	