2018 AMC 10B

Problem 1

Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

Kate 烤了一块 20 英寸 x 18 英寸的玉米面包。这块玉米面包被切分成 2 英寸 x 2 英寸的多个小块面包。问这大块玉米面包包含多少块小块面包？

$\textbf{(A)}\ 90 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 180 \qquad \textbf{(D)}\ 200 \qquad \textbf{(E)}\ 360$

Problem 2

Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

Sam 90 分钟内开了 96 英里。他在前 30 分钟内的平均速度为 60 英里每小时，在第二个 30 分钟内的平均速度为 65 英里每小时。则他在最后 30 分钟内的平均速度是多少英里每小时?

$\textbf{(A)}\ 64 \qquad \textbf{(B)}\ 65 \qquad \textbf{(C)}\ 66 \qquad \textbf{(D)}\ 67 \qquad \textbf{(E)}\ 68$

Problem 3

In the expression $(\_\_\_\_ \times \_\_\_\_) + (\_\_\_\_ \times \_\_\_\_)$ each blank is to be filled in with one of the digits $1, 2, 3,$ or $4,$ with each digit being used once. How many different values can be obtained?

在表达式$(____ \times ____) + (____ \times ____)$中，从 1，2，3 或 4 中选 1 个数字填到其中 1 个空白处，并且每个数字只能使用 1 次。可以得到多少种不同的值?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 24$

Problem 4

A three-dimensional rectangular box with dimensions X, Y, and Z has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is $X + Y + Z$?

一个三维的长方体盒子维度为 X，Y，Z，其每个面的面积分别为 24，24，48，48，72 和 72。问 $X+Y+Z$ 是多少?

(A) 18 **(B)** 22 **(C)** 24 **(D)** 30 **(E)** 36

Problem 5

How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

集合$\{2, 3, 4, 5, 6, 7, 8, 9\}$有多少个子集包含至少 1 个质数？

(A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

Problem 6

A box contains 5 chips, numbered $1, 2, 3, 4,$ and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

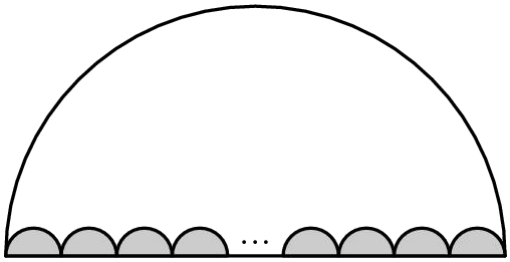
一个盒子内有 5 个芯片，编号为 1，2，3，4 和 5。一个接一个地从盒子里随机抽取芯片并且抽走的不放回，直到抽到的所有芯片的编号之和超 4。需要抽 3 次的概率是多少?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

Problem 7

In the figure below, N congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the small semicircles. The ratio $A:B$ is $1:18$. What is N?

如下图所示，沿着大的半圆的直径画了 N 个全等的小的半圆，这些小半圆的直径覆盖了大的半圆的直径且没有重叠。定义 A 为所有小半圆的面积之和，B 为大半圆内部且在所有小半圆外部的区域的面积。A:B 为 1:18。求 N 是多少?



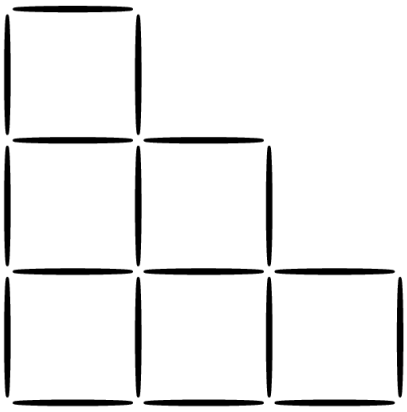
(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 36

Problem 8

Sara makes a staircase out of toothpicks as shown:

This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

Sara 用牙签做了如下图所示的楼梯，这是个三阶的楼梯，用了 18 根牙签。如果用 180 根牙签，那么楼梯是多少阶？



(A) 10 **(B)** 11 **(C)** 12 **(D)** 24 **(E)** 30

Problem 9

The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let p be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability p?

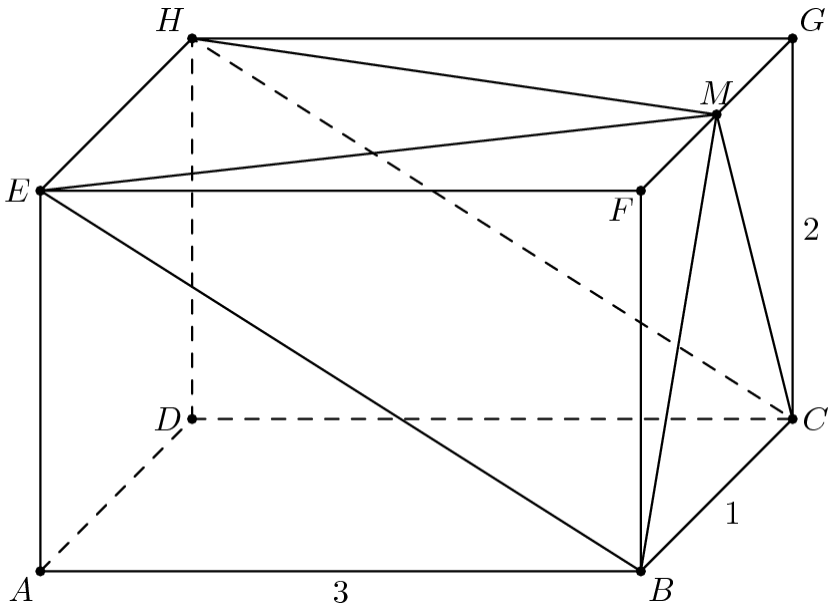
7 个标准骰子中每个骰子的 6 个面都被编号为 1 到 6。同时扔这 7 个骰子，7 个朝上的面的数字之和是 10 的概率为 p。还有其他哪个和出现的概率也是 p?

(A) 13 **(B)** 26 **(C)** 32 **(D)** 39 **(E)** 42

Problem 10

In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of $\overline{FG}$. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?

在如图所示的长方体中，*AB*=3，*BC*=1，*CG*=2。点 *M* 是 *FG* 的中点。以 *M* 为顶点，*BCHE* 为底的四棱锥的体积是多少？



(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

Problem 11

Which of the following expressions is never a prime number when p is a prime number?

当 p 是一个质数时，下面哪个表达式不可能是质数？

$\textbf{(A)}\ p^2+16 \qquad \textbf{(B)}\ p^2+24 \qquad \textbf{(C)}\ p^2+26 \qquad \textbf{(D)}\ p^2+46 \qquad \textbf{(E)}\ p^2+96$

Problem 12

Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

线段$\overline{AB}$是一个圆的直径且 AB=24。不同于 A 或 B 的点 C 在圆周上。当 C 在圆上移动时，$\triangle ABC$的重心（质心）的轨迹是一条缺少两个点的封闭曲线。和这条曲线所包围的区域的面积最接近的正整数是多少？

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 38 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 63 \qquad \textbf{(E)}\ 75$

Problem 13

How many of the first 2018 numbers in the sequence $101, 1001, 10001, 100001, \ldots$ are divisible by 101?

数列 101，1001，10001，100001...的前 2018 项，有多少项能被 101 整除？

$\textbf{(A)}\ 253 \qquad \textbf{(B)}\ 504 \qquad \textbf{(C)}\ 505 \qquad \textbf{(D)}\ 506 \qquad \textbf{(E)}\ 1009$

Problem 14

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

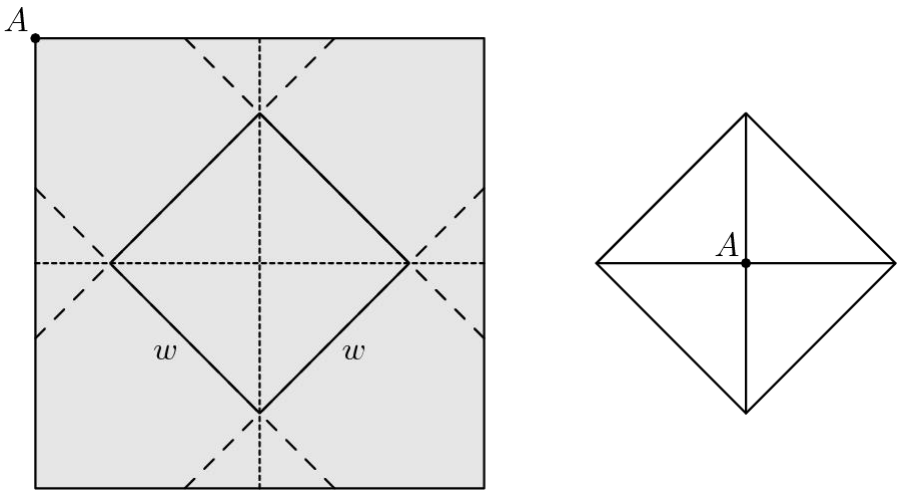
一列 2018 个正整数有唯一的众数，并且出现了恰好 10 次，这列数字中，最少可能有多少个不同的数？

$\textbf{(A)}\ 202 \qquad \textbf{(B)}\ 223 \qquad \textbf{(C)}\ 224 \qquad \textbf{(D)}\ 225 \qquad \textbf{(E)}\ 234$

Problem 15

A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?

一个底面为正方形的封闭的盒子要用一张正方形的包装纸包裹起来。这个盒子放在包装纸的中心，底面的四个顶点位于正方形包装纸的中间线上，如下图左图所示，包装纸的四个角落沿着盒子的侧面竖着折起来，并在盒子顶面的中心 A 点处重合，如下图右图所示，盒子的底面长度为 w，盒子高为 h，这张包装纸的面积为多少？



(A) $2(w+h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2+4wh$ **(D)** $2w^2$ **(E)** w^2h

Problem 16

Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that$a_1+a_2+\cdots+a_{2018}=2018^{2018}$.What is the remainder when $a_1^3+a_2^3+\cdots+a_{2018}^3$ is divided by 6?

$a_1, a_2, \ldots, a_{2018}$是严格递增的正整数数列，满足$a_1+a_2+\cdots+a_{2018}=2018^{2018}$.求$a_1^3+a_2^3+\cdots+a_{2018}^3$除以 6 所得的余数？

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 17

In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, points E and F lie on $\overline{RS}$, and points G and H lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k, m, and n are integers and n is not divisible by the square of any prime. What is $k + m + n$?

在长方形$PQRS$，*PQ*=8，*QR*=6。点 *A* 和点 *B* 在线段$\overline{PQ}$上，点 *C* 和点 *D* 在线段$\overline{QR}$上，点 *E* 和点 *F* 在线段$\overline{RS}$上，点 *G* 和点 *H* 在线段$\overline{SP}$上，满足$AP = BQ < 4$，且凸八边形 *ABCDEFGH* 是等边八边形。这个八边形的边长可以写成$k + m\sqrt{n}$，其中 *k*，*m* 和 *n* 都是整数，*n* 不能被任何质数的平方所整除。*k*+*m*+*n* 是多少？

(A) 1 **(B)** 7 **(C)** 21 **(D)** 92 **(E)** 106

Problem 18

Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

来自不同家庭的3对孩子需要坐面包车出去旅游，这6个孩子会坐在面包车的第二排和第三排，每排有 3 个位置。为了避免打扰，同一家庭的两个孩子不能坐在同一排的相邻位置，并且每个孩子也不能坐在他或她的兄弟姐妹的前面。总共有多少种安排位置的方法？

(A) 60 **(B)** 72 **(C)** 92 **(D)** 96 **(E)** 120

Problem 19

Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

Joey 和 Chloe 还有他们的女儿 Zoe 的生日是同一天。Joey 比 Chloe 大 1 岁，Zoe 今天恰好 1 岁。一共有 9 次这样的生日，生日当天 Chloe 的年龄是 Zoe 的年龄的整数倍，而今天是第 1 次。等到下次 Joey 的年龄是 Zoe 的年龄的整数倍的生日那天，Joey 年龄的 2 位数字之和是多少？

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Problem 20

A function f is defined recursively by $f(1) = f(2) = 1$ and $f(n) = f(n-1) - f(n-2) + n$ for all integers $n \geq 3$. What is $f(2018)$?

定义:$f(1) = f(2) = 1$，并且对所有整数 n≥3，$f(n) = f(n-1) - f(n-2) + n$，求 $f(2018)$。

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

Problem 21

Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

Mary 选择了一个 4 位偶数 *n*.她按照从小到大的次序依次从左向右写下 *n* 的所有因子:

$1, 2, \ldots, \frac{n}{2}, n$。在此过程中的某一时刻点，Mary 写下 *n* 的一个因子 323，那么写在 323 右边的下一个因子的最小可能值是多少?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

Problem 22

Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x, y, and 1 are the side lengths of an obtuse triangle?

实数 *x* 和 *y* 分别独立且均匀随机的从区间［0，1］中选择，那么 *x*，*y* 和 1 是一个钝角三角形的三条边的概率和下面哪个数字最接近?

(A) 0.21 **(B)** 0.25 **(C)** 0.29 **(D)** 0.50 **(E)** 0.79

Problem 23

How many ordered pairs (a, b) of positive integers satisfy the equation$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \gcd(a, b)$,where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\text{lcm}(a, b)$ denotes their least common multiple?

有多少对正整数有序对(a, b)满足方程：$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \gcd(a, b)$，这里$\gcd(a, b)$表示 a 和 b 的最大公约数，$\text{lcm}(a, b)$表示它们的最小公倍数？

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

Problem 24

Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

ABCDEF 是一个边长为 1 的正六边形。*X*，*Y* 和 *Z* 分别是边$\overline{AB}, \overline{CD},$ 和 $\overline{EF}$的中点。$\triangle ACE$和$\triangle XYZ$的内部相交所形成的凸六边形的面积是多少?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

Problem 25

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

$\lfloor x \rfloor$表示小于等于 *x* 的最大整数，有多少个实数 *x* 满足方程$x^2 + 10,000\lfloor x \rfloor = 10,000x$？

(A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201

2018 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
A	D	B	B	D	D	D	C	D	E	C	C	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	A	E	B	D	E	B	C	C	B	C	C	