2018 AMC10A

Problem 1

What is the value of

$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1?$$

下列算式的值是多少

$$\left(\left((2+1)^{-1}+1\right)^{-1}+1\right)^{-1}+1?$$

(A) $\frac{5}{8}$ **(B)** $\frac{11}{7}$ **(C)** $\frac{8}{5}$ **(D)** $\frac{18}{11}$ **(E)** $\frac{15}{8}$

Problem 2

Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have?

Liliane 有的汽水比 Jacqueline 多 50%，Alice 有的汽水比 Jacqueline 的多 25%。Liliane 和 Alice 拥有汽水量的关系是？

(A) Liliane has 20% more soda than Alice. | Liliane|有的汽水比 Alice 多 20%。

(B) Liliane has 25% more soda than Alice. | Liliane 有的汽水比 Alice 多 25%。

(C) Liliane has 45% more soda than Alice. | Liliane 有的汽水比 Alice 多 45%。

(D) Liliane has 75% more soda than Alice. | Liliane 有的汽水比 Alice 多 75%。

(E) Liliane has 100% more soda than Alice. | Liliane 有的汽水比 Alice 多 100%。

Problem 3

A unit of blood expires after $10! = 10 \cdot 9 \cdot 8 \cdots 1$ seconds. Yasin donates a unit of blood at noon of January 1. On what day does his unit of blood expire?

一个单位的血液会在$10! = 10 \cdot 9 \cdot 8 \cdots 1$秒后过期。Yasin 在一月一日中午捐献了一个单位的血液。何时他的血液会过期？

(A) January 2 **(B)** January 12 **(C)** January 22 **(D)** February 11 **(E)** February 12

Problem 4

How many ways can a student schedule 3 mathematics courses -- algebra, geometry, and number theory -- in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

一个学生想要在一天的 6 堂课中安排 3 门数学课——代数、几何和数论，要求不能连续安排任何两堂数学课（其他三堂课如何安排无需考虑），请问有多少种安排方法？

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

Problem 5

Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements were true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

Alice、Bob、和 Charlie 在徒步旅行，他们想知道离最近的城镇还有多远。Alice 说：“我们至少有 6 英里远。” Bob 回答说：“我们最多只有 5 英里远。” Charlie 评论道：“我们最多只有 4 英里远。”实际上这三个陈述都不正确，令 d 是到最近城镇的距离的英里数，以下哪个答案是 d 的所有可能值的集合？

(A) $(0, 4)$ **(B)** $(4, 5)$ **(C)** $(4, 6)$ **(D)** $(5, 6)$ **(E)** $(5, \infty)$

Problem 6

Sangho uploaded a video to a website where viewers can vote that they like or dislike a video. Each video begins with a score of 0, and the score increases by 1 for each like vote and decreases by 1 for each dislike vote. At one point Sangho saw that his video had a score of 90, and that 65% of the votes cast on his video were like votes. How many votes had been cast on Sangho's video at that point?

Sangho 上传了一个视频到某网站，观众对视频可以投出“喜欢”或“不喜欢”的票。每个视频的得分都以 0 分开始，每个“喜欢”的投票使分数增加 1，每个“不喜欢”的投票使分数减少 1。有一次，Sangho 看到他的视频有 90 分，已知 65%的投票是“喜欢”票，请问当时 Sangho 的视频共得到了多少票？

$\textbf{(A)}\ 200 \qquad \textbf{(B)}\ 300 \qquad \textbf{(C)}\ 400 \qquad \textbf{(D)}\ 500 \qquad \textbf{(E)}\ 600$

Problem 7

For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\tfrac{2}{5}\right)^n$ an integer?

n（不一定是正数）有多少种取值方式使得$4000 \cdot \left(\tfrac{2}{5}\right)^n$的结果是一个整数？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 8

Joe has a collection of 23 coins, consisting of 5-cent coins, 10-cent coins, and 25-cent coins. He has 3 more 10-cent coins than 5-cent coins, and the total value of his collection is 320 cents. How many more 25-cent coins does Joe have than 5-cent coins?

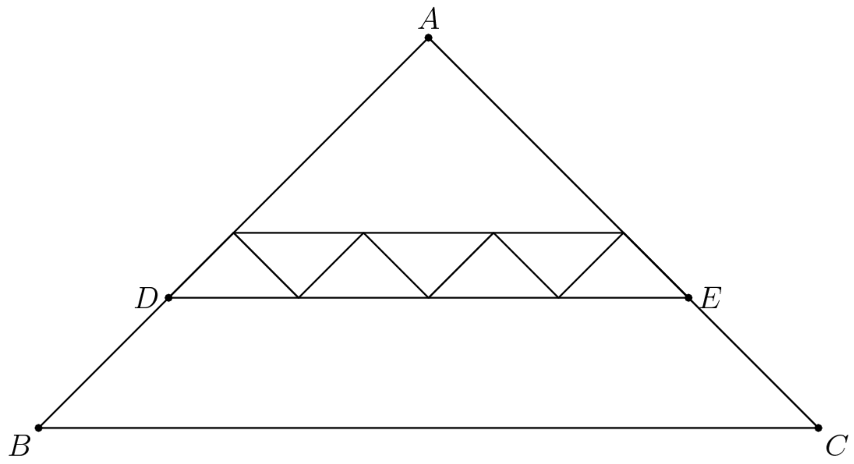
Joe 收集了 23 枚硬币，由面值 5 美分，10 美分和 25 美分的硬币组成。他的 10 美分面值硬币 比 5 美分面值硬币多 3 枚，他的硬币收藏总价值为 320 美分。他的 25 美分面值的硬币比 5 美分面值硬币多了多少枚？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 9

All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?

下图中的所有三角形与等腰三角形 ABC 类似，$AB = AC$。 7 个最小的三角形的面积都是 1，$\triangle ABC$的面积是 40。梯形 $DBCE$ 的面积是多少？



(A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

Problem 10

Suppose that real number x satisfies $\sqrt{49-x^2}-\sqrt{25-x^2}=3$.What is the value of $\sqrt{49-x^2}+\sqrt{25-x^2}$?

假设实数 x 满足$\sqrt{49-x^2}-\sqrt{25-x^2}=3$.那么$\sqrt{49-x^2}+\sqrt{25-x^2}$的值是多少？

(A) 8 **(B)** $\sqrt{33}+8$ **(C)** 9 **(D)** $2\sqrt{10}+4$ **(E)** 12

Problem 11

When 7 fair standard 6-sided dice are thrown, the probability that the sum of the numbers on the top faces is 10 can be written as $\frac{n}{6^7}$,where n is a positive integer. What is n?

当掷出 7 枚标准的 6 个面的骰子时，朝上的面上的各个数相加总和为 10 的概率可以写成$\frac{n}{6^7}$，其中 n 是一个正整数。问 n 是多少？

$\textbf{(A)}\ 42 \qquad \textbf{(B)}\ 49 \qquad \textbf{(C)}\ 56 \qquad \textbf{(D)}\ 63 \qquad \textbf{(E)}\ 84$

Problem 12

How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$\begin{align*} x+3y&=3 \\ \big||x|-|y|\big|&=1 \end{align*}$$

有多少个有序实数对(x, y)满足下面的方程组？

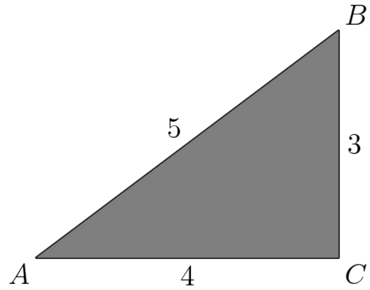
$$\begin{align*} x+3y&=3 \\ \big||x|-|y|\big|&=1 \end{align*}$$

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 8$

Problem 13

A paper triangle with sides of lengths $3, 4,$ and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?

如图所示，三边长度为 3、4 和 5 英寸的三角形纸被折叠，使得点 A 落在点 B 上。折痕的长度是多少英寸？



$\textbf{(A)}\ 1+\frac{1}{2}\sqrt{2} \qquad \textbf{(B)}\ \sqrt{3} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ \frac{15}{8} \qquad \textbf{(E)}\ 2$

Problem 14

What is the greatest integer less than or equal to $\dfrac{3^{100}+2^{100}}{3^{96}+2^{96}}$?

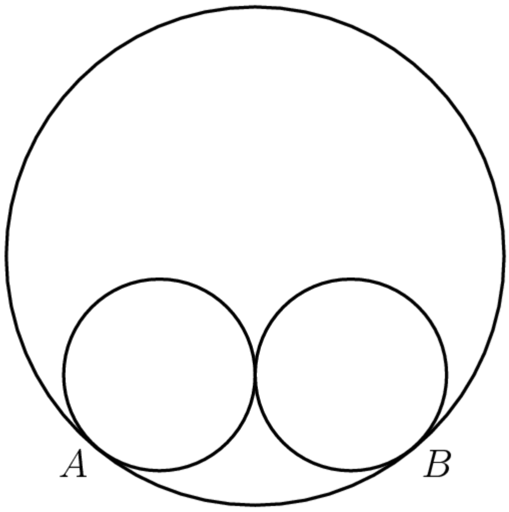
小于或等于 $\dfrac{3^{100}+2^{100}}{3^{96}+2^{96}}$ 的最大整数是多少？

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 81 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 97 \qquad \textbf{(E)}\ 625$

Problem 15

Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. The distance AB can be written in the form $\tfrac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

如图所示，半径为 5 的两个圆互相外切，并且在点 A 和 B 与半径为 13 的圆内切。距离 AB 可以写成 $\tfrac{m}{n}$ 的形式，其中 m 和 n 是互质的两个正整数。 $m+n$ 是多少？



$\textbf{(A)}\ 21 \qquad \textbf{(B)}\ 29 \qquad \textbf{(C)}\ 58 \qquad \textbf{(D)}\ 69 \qquad \textbf{(E)}\ 93$

Problem 16

Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$?

直角三角形 ABC 的直角边长 $AB = 20$，$BC = 21$。包括 AB 和 BC，从顶点 B 到斜边 AC 可以绘制多少条长度为整数的线段？

(A) 5 **(B)** 8 **(C)** 12 **(D)** 13 **(E)** 15

Problem 17

Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element in S?

设 S 是从$\{1, 2, \cdots, 12\}$中取出 6 个整数组成的集合，具有性质：如果 a 和 b 是 S 中的元素并满足 $a < b$，那么 b 不是 a 的倍数。 S 中元素的最小可能取值是多少？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

Problem 18

How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

有多少个非负整数可以写成以下形式

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

其中对于$0 \le i \le 7$，有$a_i \in \{-1, 0, 1\}$？

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** $59,048$

Problem 19

A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number n is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

m 是从集合{11, 13, 15, 17, 19}中任意选取的一个数, *n* 是从集合{1999, 2000, 2001, ⋯ , 2018}中任意选取的一个数。m^n的个位数字为 1 的概率是多少?

$\textbf{(A)}\ \frac{1}{5} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{3}{10} \qquad \textbf{(D)}\ \frac{7}{20} \qquad \textbf{(E)}\ \frac{2}{5}$

Problem 20

A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called $\textit{symmetric}$ if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

一个扫描代码由一个 7 x 7 的正方形网格组成，其中一些正方形的颜色为黑色，其余为白色。这 49 格子里每种颜色的正方形至少有一个。如果整个扫描码以中心为轴逆时针旋转$90°$的倍数，或是沿着正方形的对角线以及连接对边中点的联机翻转时，整个图形都不改变，那么这个扫面代码被称为**对称的**。所有可能的对称扫描代码的总数是多少?

$\textbf{(A)}\ 510 \qquad \textbf{(B)}\ 1022 \qquad \textbf{(C)}\ 8190 \qquad \textbf{(D)}\ 8192 \qquad \textbf{(E)}\ 65,534$

Problem 21

Which of the following describes the set of values of a for which the curves $x^2+y^2=a^2$ and $y=x^2-a$ in the real xy-plane intersect at exactly 3 points?

使得曲线$x^2+y^2=a^2$和$y=x^2-a$在 xy 实坐标系恰好有 3 个交点的 a 的取值范围是什么?

$\textbf{(A)}\ a=\frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{4}< a< \frac{1}{2} \qquad \textbf{(C)}\ a>\frac{1}{4} \qquad \textbf{(D)}\ a=\frac{1}{2} \qquad \textbf{(E)}\ a>\frac{1}{2}$

Problem 22

Let $a, b, c,$ and d be positive integers such that $\gcd(a, b) = 24$, $\gcd(b, c) = 36$, $\gcd(c, d) = 54$, and $70 < \gcd(d, a) < 100$. Which of the following must be a divisor of a?

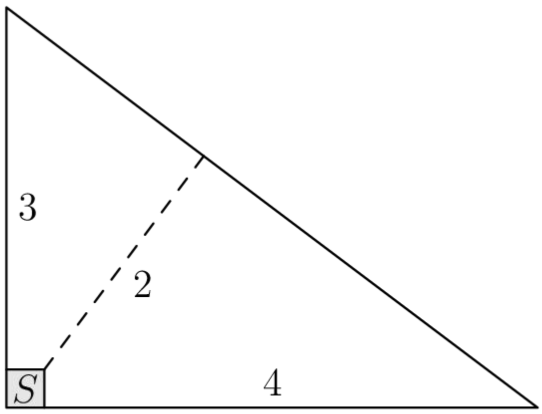
a，b，c 和 d 是正整数， 并且满足 gcd(a, b) = 24, gcd(b, c) = 36, gcd(c, d) = 54 和 70 < gcd(d, a)< 100. 以下哪个数一定是 a 的约数? （备注：对于正整数 x，y，gcd(x, y)表示 x 和 y 的最大公约数。）

(A) 5 **(B)** 7 **(C)** 11 **(D)** 13 **(E)** 17

Problem 23

Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?

农场主 Pythagoras 有一块形状为直角三角形的土地，两条直角边的长度分别为 3 个单位和 4 个单位。在两条直角边相交形成直角的角落处，他留下了一小块正方形土地没有进行种植，如图标记为 S 的区域，使得从空中看下去很像是直角的标识。其余的土地都进行了种植。从 S 到斜边的最短距离为 2 个单位。整块土地的种植比例是多少？



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

Problem 24

Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

在三角形 ABC 中，$AB = 50, AC = 10$，其面积为 120。令 D 为 AB 的中点并且 E 为 AC 的中点。∠BAC 的角平分线分别与 DE 和 BC 相交于 F 和 G。四边形 $FDBG$ 的面积是多少？

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

Problem 25

For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b, and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

对于正整数 n 和非零数字 a, b 以及 c，令A_n为 n 位整数，其每一位数字都等于 a；令B_n为 n 位整数，其每一位数字都等于 b；令C_n为 $2n$ 位整数（非 n 位整数），其每一位数字都等于 c。使得等式$C_n - B_n = A_n^2$对于至少两个 n 值成立的 $a + b + c$ 最大可能取值为多少？

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

2018 AMC10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
B	A	E	E	D	B	E	C	E	A	E	C	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
A	D	D	C	D	E	B	E	D	D	D	D	