2017 AMC 10B

Problem 1

Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number?

Mary 想出了一个 2 位正数。她把它乘以 3 再加上 11.之后把所得结果的两位数字交换位置，得到了一个在 71 和 75 之间的数（包含 71 和 75）。Mary 原来想出的数是多少?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Problem 2

Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps?

Sofia 沿着她学校的 400 米跑道跑了 5 圈。她每一圈前 100 米的平均速度是 4 米每秒，剩余 300 米的平均速度是 5 米每秒，那么 Sofia 跑 5 圈总共花了多少时间？

（A） 5 minutes and 35 seconds丨5 分钟 35 秒

（B） 6 minutes and 40 seconds丨6 分钟 40 秒

（C） 7 minutes and 5 seconds丨7 分钟 5 秒

（D） 7 minutes and 25 seconds丨7 分钟 25 秒

（E）8 minutes and 10 seconds 丨8 分钟 10 秒

Problem 3

Real numbers x, y, and z satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

实数 x，y，z 满足不等式 $0 < x < 1$, $-1 < y < 0$ 和 $1 < z < 2$,下面哪个数字是正数?

$\textbf{(A)}\ y + x^2 \qquad \textbf{(B)}\ y + xz \qquad \textbf{(C)}\ y + y^2 \qquad \textbf{(D)}\ y + 2y^2 \qquad \textbf{(E)}\ y + z$

Problem 4

Suppose that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y} = -2$. What is the value of $\frac{x+3y}{3x-y}$?

假设 x 和 y 是非零实数，满足 $\frac{3x+y}{x-3y} = -2$。则 $\frac{x+3y}{3x-y}$ 的值为多少？

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Problem 5

Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

Camilla 原先拥有的蓝莓味软心豆粒糖的数量是樱桃味软心豆粒糖的 2 倍。当她 2 种口味都分别吃了 10 块后，现在她的蓝莓味软心豆粒糖的数量是樱桃味软心豆粒糖的 3 倍。那么她原先有多少块蓝莓味软心豆粒糖?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 50$

Problem 6

What is the largest number of solid 2 in. by 2 in. by 1 in. blocks that can fit in a 3 in. by 2 in. by 3 in. box?

将2 in.乘2 in.乘1 in.的方块放入3 in.乘2 in.乘3 in.的盒子里，最多可以装多少块？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 7

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

Samia 骑着自行车去访问她的朋友，平均速度为 17 公里每小时，当她走了一半的路程，一个轮胎坏了，之后她以 5 公里每小时的速度步行走完了剩余的路程，最终到她朋友家总共花了 44 分钟。Samia 步行了多少公里路程？结果保留一位小数。

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Problem 8

Points $A(11,9)$ and $B(2,-3)$ are vertices of $\triangle ABC$ with $AB=AC$. The altitude from A meets the opposite side at $D(-1,3)$. What are the coordinates of point C?

点 A（11，9）和点 B（2，−3）是$\triangle ABC$的顶点，且 $AB=AC$。从 A 引出的高和对边交于 D（−1，3）。点 C 的坐标为多少?

$\textbf{(A)}\ (-8,9) \qquad \textbf{(B)}\ (-4,8) \qquad \textbf{(C)}\ (-4,9) \qquad \textbf{(D)}\ (-2,3) \qquad \textbf{(E)}\ (-1,0)$

Problem 9

A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

一个广播节目有一项由 3 道选择题组成的测试，每道选择题有 3 个选项．参赛者若能做对 2 道或者更多道题目，那么他或她就赢了，参赛者每道选择题都是随机作答。那么赢得测试的概率是多少？

$\textbf{(A)}\ \frac{1}{27} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{2}{9} \qquad \textbf{(D)}\ \frac{7}{27} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 10

The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is c?

两条直线$ax - 2y = c$和$2x + by = -c$互相垂直且交于点（1，-5）。c 是多少?

$\textbf{(A)}\ -13 \qquad \textbf{(B)}\ -8 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 13$

Problem 11

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

在 Typico 高中，60％的学生喜欢跳舞，剩余的人不喜欢。在那些喜欢跳舞的人中，80％的人说他们喜欢跳舞，剩余的人说他们不喜欢跳舞，而在那些不喜欢跳舞的人中，90％的人说他们不喜欢跳舞，其余的人说他们喜欢跳舞，那些说自己不喜欢跳舞的学生当中，求实际喜欢跳舞的人占比多少？

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ 33\frac{1}{3}\%$

Problem 12

Elmer's new car gives 50% better fuel efficiency. However, the new car uses diesel fuel, which is 20% more expensive per liter than the gasoline the old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

Elmer 的新车的燃油效率提高了 50％，单位是公里每升。然而，新车使用的是柴油，每升的价格比旧车使用的汽油高了 20％。若他使用新车代替旧车进行长途旅行，他将省百分之多少的钱？

$\textbf{(A)}\ 20\% \qquad \textbf{(B)}\ 26\frac{2}{3}\% \qquad \textbf{(C)}\ 27\frac{7}{9}\% \qquad \textbf{(D)}\ 33\frac{1}{3}\% \qquad \textbf{(E)}\ 66\frac{2}{3}\%$

Problem 13

There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

一共有 20 个学生参加一个提供瑜伽，桥牌和绘画这 3 门课程的课外培训。每个学生都必须参加 3 门课程中至少一门，但也可以同时参加 2 门或全部 3 门。一共有 10 个学生参加瑜伽，13 个学生参加桥牌，9 个学生参加绘画。有 9 个学生参加了至少 2 门课，有多少个学生参加了全部 3 门课？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 14

An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1?

从 1≤*N*≤2020 的范围内随机选择一个整数 *N*。则N^{16}除以 5 余数是 1 的概率是多少？

$\textbf{(A)}\ \frac{1}{5} \qquad \textbf{(B)}\ \frac{2}{5} \qquad \textbf{(C)}\ \frac{3}{5} \qquad \textbf{(D)}\ \frac{4}{5} \qquad \textbf{(E)}\ 1$

Problem 15

Rectangle $ABCD$ has $AB = 3$ and $BC = 4$. Point E is the foot of the perpendicular from B to diagonal $\overline{AC}$. What is the area of $\triangle AED$?

长方形 *ABCD* 中， *AB*=3，*BC*=4，过 *B* 向对角线$\overline{AC}$作垂线，垂足为 *E*。$\triangle AED$的面积是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{42}{25} \qquad \textbf{(C)}\ \frac{28}{15} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{54}{25}$

Problem 16

How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

小于或等于 2017 的所有 10 进制正整数中，有多少个含有数字 0?

$\textbf{(A)}\ 469 \qquad \textbf{(B)}\ 471 \qquad \textbf{(C)}\ 475 \qquad \textbf{(D)}\ 478 \qquad \textbf{(E)}\ 481$

Problem 17

Call a positive integer **monotonous** if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

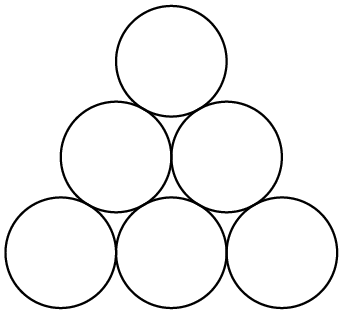
如果一个正整数是一位数字或者当从左往右读时，它的各个位上的数字形成一个严格递增或严格递减的数列，那么我们把此正整数称作单调的，例如，3,23578 和 987620 都是单调的，但是 88,7434，和 23557 不是，一共有多少个单调的正整数？

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

Problem 18

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?

如下图，6 个圆盘中其中 3 个涂成蓝色，2 个涂成红色，1 个涂成绿色。如果两种涂色图案可以通过旋转或者对称重合，那么这 2 种涂色图案视作同一个。一共有多少种不同的图案?



$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Problem 19

Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3 \cdot CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

ABC 为一个等边三角形。延长边$\overline{AB}$到 B' 使得 BB' =3AB。类似的，延长边$\overline{BC}$到 C' 使得 CC' =3BC，延长边$\overline{CA}$到 A' 使得 AA' =3CA。则$\triangle A'B'C'$的面积和$\triangle ABC$的面积的比值是多少？

(A) $9:1$ **(B)** $16:1$ **(C)** $25:1$ **(D)** $36:1$ **(E)** $37:1$

Problem 20

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

数字 21！=51,090,942,171,709,440,000 的正整数因子的个数超过 60,000 个。从这些因于中随机选择 1 个，这个数是奇数的概率是多少？

(A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

Problem 21

In $\triangle ABC$, $AB = 6$, $AC = 8$, $BC = 10$, and D is the midpoint of $\overline{BC}$. What is the sum of the radii of the circles inscribed in $\triangle ADB$ and $\triangle ADC$?

在$\triangle ABC$中，AB=6，AC=8，BC=10，且 D 为$\overline{BC}$的中点，那么$\triangle ADB$和$\triangle ADC$的内切圆的半径之和为多少？

(A) $\sqrt{5}$ **(B)** $\frac{11}{4}$ **(C)** $2\sqrt{2}$ **(D)** $\frac{17}{6}$ **(E)** 3

Problem 22

The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD.
Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

半径为 2 的圆的直径 *AB* 延长到位于圆外的点 *D*，使得 *BD*=3。选择一个点 *E*，使得 *ED*=5 且 *ED* 垂直于 *AD*。线段 *AE* 和圆的交点 *C* 位于 *A* 和 *E* 之间。$\triangle ABC$的面积为多少？

$\textbf{(A)}\ \frac{120}{37} \qquad \textbf{(B)}\ \frac{140}{39} \qquad \textbf{(C)}\ \frac{145}{39} \qquad \textbf{(D)}\ \frac{140}{37} \qquad \textbf{(E)}\ \frac{120}{31}$

Problem 23

Let $N = 123456789101112\ldots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

N=123456789101112...4344 是 1 个 79 位的数字，是由从 1 依次写到 44 的整数形成。*N* 除以 45 所得余数为多少?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 44$

Problem 24

The vertices of an equilateral triangle lie on the hyperbola $xy = 1$, and a vertex of this hyperbola is the centroid of the triangle. What is the square of the area of the triangle?

一个等边三角形的顶点位于双曲线 *xy*=1 上。这个双曲线的一个顶点是三角形的重心。这个三角形的面积的平方是多少？

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 108 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 169$

Problem 25

Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

去年 Isabella 参加了 7 场数学考试并得到 7 个分数，每个分数都是 91 到 100 之间且包括 91 和 100 的整数。每场考试后，她发现她考试的平均分都是个整数。她第 7 场考试的分数是 95 分。那么她第 6 场考试得分是多少?

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 94 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 98 \qquad \textbf{(E)}\ 100$

2017 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
B	C	E	D	D	B	C	C	D	E	D	A	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	E	A	B	D	E	B	D	D	C	C	E	