2016 AMC 10A

Problem 1

What is the value of $\dfrac{11!-10!}{9!}$?

$\dfrac{11!-10!}{9!}$ 的值是多少？

$\textbf{(A)}\ 99 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 110 \qquad \textbf{(D)}\ 121 \qquad \textbf{(E)}\ 132$

Problem 2

For what value x does $10^{x}\cdot 100^{2x}=1000^{5}$?

$10^{x}\cdot 100^{2x}=1000^{5}$，求 x。

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 3

For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid $\$12.50$ more than David. How much did they spend in the bagel store together?

Ben 在硬面包上每花费 1 美元，David 就比他少花 25 美分。Ben 总共比 David 多花了 12.5 美元，那么他们在面包店里一起总共花了多少钱？

$\textbf{(A)}\ \$37.50 \qquad \textbf{(B)}\ \$50.00 \qquad \textbf{(C)}\ \$87.50 \qquad \textbf{(D)}\ \$90.00 \qquad \textbf{(E)}\ \$92.50$

Problem 4

The remainder can be defined for all real numbers x and y with $y \neq 0$ by $\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor$ where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$?

对于所有实数 x 和 y 定义余数为：$\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor$，其中$y \neq 0$，且$\left\lfloor \frac{x}{y} \right\rfloor$表示取小于或等于$\frac{x}{y}$ 的最大整数，求$\text{rem}\left(\frac{3}{8}, -\frac{2}{5}\right)$。

(A) $-\frac{3}{8}$ **(B)** $-\frac{1}{40}$ **(C)** 0 **(D)** $\frac{3}{8}$ **(E)** $\frac{31}{40}$

Problem 5

A rectangular box has integer side lengths in the ratio $1:3:4$.Which of the following could be the volume of the box?

一个边长是整数的长方体盒子的三边之比为 1:3:4。下面哪个可能是盒子的体积?

(A) 48 **(B)** 56 **(C)** 64 **(D)** 96 **(E)** 144

Problem 6

Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?

Ximena 把 1 到 30 的整数列成一列，Emilio 拷贝了 Ximena 的数字，然后把所有的数字 2 替换成数字 1。Ximena 把她自己的所有数相加，同时 Emilio 也把他自己的数相加。Ximena 得到的和比 Emilio 得到的和大多少？

(A) 13 **(B)** 26 **(C)** 102 **(D)** 103 **(E)** 110

Problem 7

The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

7 个数字 60，100，x，40，50，200，90 的平均值，中位数和众数都等于 x。x 是多少?

$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 75 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 100$

Problem 8

Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling, Fox is excited about his good fortune until he discovers that all his money is gone after crossing the bridge three times. How many coins did Fox have at the beginning?

诡计多端的兔子和愚蠢的狐狸有这样一个约定：每次狐狸经过兔子家门前的桥时，狐狸只要支付 40 个硬币的过桥费，兔子就可以让狐狸的硬币数翻倍。先后顺序规定兔子先让狐狸的硬币总数翻倍，然后狐狸给兔子过桥费。狐狸对于这个发财机会高兴极了，直到过桥 3 次后，它发现它的钱都没了，那么一开始狐狸有多少枚硬币？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 40 \qquad \textbf{(E)}\ 45$

Problem 9

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N?

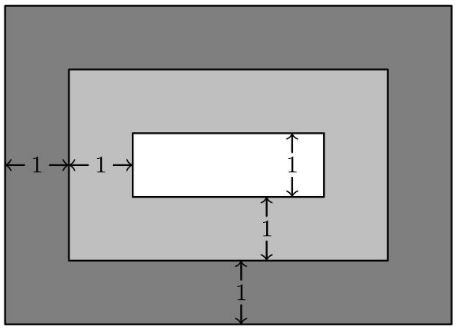
含有总共 2016 个硬币的三角形阵列的第一行有 1 个硬币，第二行有 2 个硬币，第三行有 3 个硬币，以此类推直到第 N 行有 N 个硬币。N 的各个位上数字之和为多少?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 10

A rug is made with three different colors as shown. The areas of the three differently colored regions form an arithmetic progression. The inner rectangle is one foot wide, and each of the two shaded regions is 1 foot wide on all four sides. What is the length in feet of the inner rectangle?

一块布由三种不同的颜色做成，如图所示.三块不同颜色区域的面积形成一个等差数列。最里面的长方形的宽是 1 英尺，两个阴影部分在四条边上的宽度也都是 1 英尺。最里面的长方形的长是多少英尺?

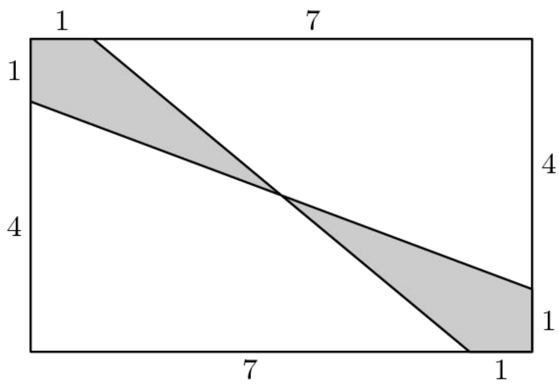


$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 11

Find the area of the shaded region.

下面长方形的阴影部分的面积是多少？



$\textbf{(A)}\ 4\frac{3}{5} \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 5\frac{1}{4} \qquad \textbf{(D)}\ 6\frac{1}{2} \qquad \textbf{(E)}\ 8$

Problem 12

Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability p that the product of the three integers is odd?

从 1 到 2016 之间（包含 1 和 2016）随机选择 3 个不同的整数。p 表示这 3 个整数的乘积是个奇数的概率，那么下面关于 p 的说法哪个是正确的？

$\textbf{(A)}\ p<\dfrac{1}{8}\qquad\textbf{(B)}\ p=\dfrac{1}{8}\qquad\textbf{(C)}\ \dfrac{1}{8}<p<\dfrac{1}{3}\qquad\textbf{(D)}\ p=\dfrac{1}{3}\qquad\textbf{(E)}\ p>\dfrac{1}{3}$

Problem 13

Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

5 个朋友在电影院里坐在一排从左向右编号为 1 到 5 的 5 个座位上。（这里的“左”和“右”是指当这 5 人坐在座位上，从他们的角度看到的左和右）在看电影的过程中，Ada 去休息室拿了一些爆米花，当她回来时，她发现 Bea 已经向右挪动了 2 个位置，Ceci 向左挪动了一个位置，Dee 和 Edie 交换了位置，空了一张最边上的位置给 Ada。那么 Ada 在她起身离开前，原先是坐在哪个座位的？

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ 3\qquad\textbf{(D)}\ 4\qquad\textbf{(E)}\ 5$

Problem 14

How many ways are there to write 2016 as the sum of twos and threes, ignoring order? (For example, $1008 \cdot 2 + 0 \cdot 3$ and $402 \cdot 2 + 404 \cdot 3$ are two such ways.)

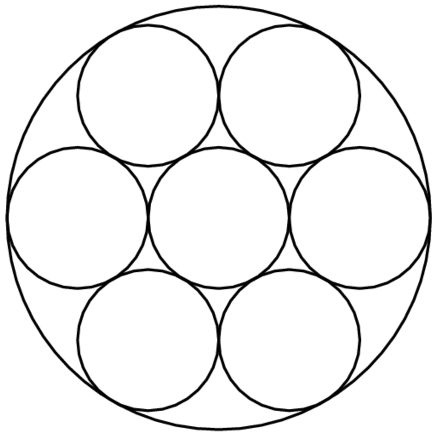
把 2016 写成若干个 2 和若干个 3 的和，不考虑顺序，有多少种方法？

$\textbf{(A)}\ 236\qquad\textbf{(B)}\ 336\qquad\textbf{(C)}\ 337\qquad\textbf{(D)}\ 403\qquad\textbf{(E)}\ 672$

Problem 15

Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?

如图所示从一个圆形的曲奇生面团上切出七个半径为 1 英寸的圆形曲奇.相邻的曲奇相切，除了中间的那个曲奇，其他曲奇都和生面团的边缘相切。切掉后剩下的边角料还可以重新揉成一个厚度一样的圆形曲奇。问剩余边角料做成的圆形曲奇的半径是多少？



(A) $\sqrt{2}$ **(B)** 1.5 **(C)** $\sqrt{\pi}$ **(D)** $\sqrt{2\pi}$ **(E)** π

Problem 16

A triangle with vertices $A(0,2)$, $B(-3,2)$, and $C(-3,0)$ is reflected about the x-axis, then the image $\triangle A'B'C'$ is rotated counterclockwise about the origin by 90° to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

一个顶点为 A(0,2)，B(-3,2)，C(-3,0)的三角形首先关于 x 轴作对称，得到的对称图像$\triangle A'B'C'$，再绕着原点逆时针旋转 90° 得到$\triangle A''B''C''$。下面哪个变换可以把$\triangle A''B''C''$再变回原来的三角形$\triangle A'B'C'$？

（A）counterclockwise rotation about the origin by 90° ｜绕着原点逆时针旋转 90°。

（B）clockwise rotation about the origin by 90° ｜绕着原点顺时针旋转 90°。

（C）reflection about the x-axis｜关于 x 轴对称。

（D）reflection about the line y=x｜关于直线 y=x 对称。

（E） reflection about the y-axis｜关于轴对称。

Problem 17

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

N 是一个正整数且是 5 的倍数。一个红球和 N 个绿球被随机排成一排。$P(N)$表示至少 $\frac{3}{5}$ 的绿球在红球的同一边的概率。注意到，$P(5) = 1$，且当 N 很大时，$P(N)$接近$\frac{4}{5}$.满足的最小的 N 的各个位上数字之和是多少？

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

Problem 18

Each vertex of a cube is to be labeled with an integer 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

一个立方体的每个顶点被 1 到 8 这 8 个整数标记，且每个整数只能使用 1 次，满足每个面的 4 个顶点上的数字之和都相同，两种排列方法如果可以通过旋转立方体变成同一种，那么这两种排列方法被认为是同一种方法。一共有多少种不同的排列方法？

(A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

Problem 19

In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point E between B and C, and point F between E and C are such that $BE = EF = FC$. Segments $\overline{AE}$ and $\overline{AF}$ intersect $\overline{BD}$ at P and Q, respectively. The ratio $BP : PQ : QD$ can be written as $r : s : t$ where the greatest common factor of r, s and t is 1. What is $r + s + t$?

在长方形 $ABCD$ 中，$AB=6$，$BC=3$。点 E 在点 B 和 C 之间，点 F 在点 E 和 C 之间，满足 $BE=EF=FC$。线段 $\overline{AE}$ 和$\overline{AF}$分别交$\overline{BD}$于点 P 和 Q。$BP : PQ : QD$的比值可以写成$r : s : t$，其中 r，s 和 t 的最大公约数是 1。$r+s+t$ 是多少？

(A) 7 **(B)** 9 **(C)** 12 **(D)** 15 **(E)** 20

Problem 20

For some particular value of N, when $(a+b+c+d+1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables $a, b, c,$ and d, each to some positive power. What is N?

对于某个特定的 N 值，当多项式$(a+b+c+d+1)^N$被展开，同类项被合并后，得到的结果表达式中，变量 a，b，c 和 d 的指数是正数的项一共有 1001 项。N 是多少?

$\textbf{(A) } 9 \qquad \textbf{(B) } 14 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 19$

Problem 21

Circles with centers P, Q and R, having radii $1, 2$ and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

圆心为 P，Q 和 R 的圆半径分别为 1，2，3，它们在直线l的同一侧，且和l相切，切点分别为 P', Q' a 和R'，其中 Q' 位于 P' 和 R' 之间。圆心为 Q 的圆和另外两个圆外切。三角形 PQR 的面积是多少？

$\textbf{(A) } 0 \qquad \textbf{(B) } \sqrt{6}/3 \qquad \textbf{(C) } 1 \qquad \textbf{(D) } \sqrt{6}-\sqrt{2} \qquad \textbf{(E) } \sqrt{6}/2$

Problem 22

For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

n 是个正整数，数字$110n^3$有 110 个正整数因子，包括 1 和$110n^3$。那么数字$81n^4$有多少个正整数因子?

$\textbf{(A) } 110 \qquad \textbf{(B) } 191 \qquad \textbf{(C) } 261 \qquad \textbf{(D) } 325 \qquad \textbf{(E) } 425$

Problem 23

A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers $a, b,$ and c. (Here . represents multiplication). The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q?$

二元运算符$\diamondsuit$有这样的性质:对于所有的非零实数 a ，b，c，有$a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$，且 $a \diamondsuit a = 1$（这里 • 表示乘法）。方程$2016 \diamondsuit (6 \diamondsuit x) = 100$的解可写成 $\frac{p}{q}$ ，其中 p 和 q 是互质的正整数。求 $p+q$ 的值？

(A) 109 **(B)** 201 **(C)** 301 **(D)** 3049 **(E)** 33, 601

Problem 24

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of the fourth side?

一个四边形内接在半径为$200\sqrt{2}$的圆内，四边形的三条边长度都是 200，那么第四条边长度是多少？

(A) 200 **(B)** $200\sqrt{2}$ **(C)** $200\sqrt{3}$ **(D)** $300\sqrt{2}$ **(E)** 500

Problem 25

How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72, \text{lcm}(x, z) = 600$ and $\text{lcm}(y, z) = 900$?

有多少个这样的有序正整数三元组 (x, y, z)，满足$\text{lcm}(x, y) = 72, \text{lcm}(x, z) = 600$和 $\text{lcm}(y, z) = 900$？

(A) 15 **(B)** 16 **(C)** 24 **(D)** 27 **(E)** 64

2016 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
B	C	C	B	D	D	D	C	D	B	D	A	B
14	15	16	17	18	19	20	21	22	23	24	25	
C	A	D	A	C	E	B	D	D	A	E	A	