2014 AMC 10A

Problem 1

What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?

$10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?的值是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ \frac{25}{2} \qquad \textbf{(D)}\ \frac{170}{3} \qquad \textbf{(E)}\ 170$

Problem 2

Roy's cat eats $\frac{1}{3}$ of a can of cat food every morning and $\frac{1}{4}$ of a can of cat food every evening. Before feeding his cat on Monday morning, Roy opened a box containing 6 cans of cat food. On what day of the week did the cat finish eating all the cat food in the box?

Roy 的猫每天早上吃一罐猫粮的$\frac{1}{3}$，每天晚上吃一罐猫粮的$\frac{1}{4}$。某个周一的早上，在 Roy 开始喂猫之前，他打开了一个盒子，里面有 6 罐猫粮。猫会在周几吃完盒子里所有的猫粮?

$\textbf{(A)}\ \text{Tuesday} \qquad \textbf{(B)}\ \text{Wednesday} \qquad \textbf{(C)}\ \text{Thursday} \qquad \textbf{(D)}\ \text{Friday} \qquad \textbf{(E)}\ \text{Saturday}$

Problem 3

Bridget bakes 48 loaves of bread for her bakery. She sells half of them in the morning for $\$2.50$ each. In the afternoon she sells two thirds of what she has left, and because they are not fresh, she charges only half price. In the late afternoon she sells the remaining loaves at a dollar each. Each loaf costs $\$0.75$ for her to make. In dollars, what is her profit for the day?

Bridget 为她的面包店烤了 48 片面包，她早上以每片 2.5 美元的价格卖出去了一半，下午的时候，她卖出去了剩下的三分之二，而因为面包不新鲜了，所以她以半价出售．傍晚时，她以每片 1 美元的价格把剩下的都卖了出去，已知每片面包的成本是 0.75 美元，问她这天的利润是多少美元？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 44 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 52$

Problem 4

Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

当 Ralph 走过 Jane 街时，她经过了一排 4 座楼房，每座楼房都涂有不同的颜色。她先经过桔色的房子，再经过红色房子。她先经过蓝色房子，再经过黄色房子。蓝色房子不和黄色房子相邻，这 4 个涂色的房子有多少种可能的排列方法？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 5

On an algebra quiz, 10% of the students
scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

在一次代数测试中，10%的学生得了 70 分，35%的学生得 80 分，30%的学生得 90 分，剩下的得了 100 分，问这次考试学生分数的平均数和中位数的差是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 6

Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

假设 a 头牛 c 天内产出 b 加仑的牛奶。以这样的速度，d 头牛在 e 天内会产出多少加仑的牛奶？

$\textbf{(A)}\ \frac{bde}{ac} \qquad \textbf{(B)}\ \frac{ac}{bde} \qquad \textbf{(C)}\ \frac{abde}{c} \qquad \textbf{(D)}\ \frac{bcde}{a} \qquad \textbf{(E)}\ \frac{abc}{de}$

Problem 7

Nonzero real numbers x, y, a, and b satisfy $x < a$ and $y < b$. How many of the following inequalities must be true?

非零实数 x，y，a，b 满足$x < a$且$y < b$。下面的不等式中，有多少个一定正确？

(I) $x + y < a + b$

(II) $x - y < a - b$

(III) $xy < ab$

(IV) $\frac{x}{y} < \frac{a}{b}$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 8

Which of the following numbers is a perfect square?

下面哪个数是完全平方?

(A) $\frac{14!15!}{2}$ **(B)** $\frac{15!16!}{2}$ **(C)** $\frac{16!17!}{2}$ **(D)** $\frac{17!18!}{2}$ **(E)** $\frac{18!19!}{2}$

Problem 9

The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

一个直角三角形的两条直角边，同时也是高，长度分别是$2\sqrt{3}$和 6。问这个三角形的第三条高是多长?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Problem 10

Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

以 a 开头的 5 个连续的正整数的平均值是 b，那么以 b 开头的 5 个连续的整数的平均值是多少？

$\textbf{(A)}\ a+3 \qquad \textbf{(B)}\ a+4 \qquad \textbf{(C)}\ a+5 \qquad \textbf{(D)}\ a+6 \qquad \textbf{(E)}\ a+7$

Problem 11

A customer who intends to purchase an appliance has three coupons, only one of which may be used:

Coupon 1: 10% off the listed price if the listed price is at least $\$50$

Coupon 2: $\$20$ off the listed price if the listed price is at least $\$100$

Coupon 3: 18% off the amount by which the listed price exceeds $\$100$

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

一位想要买家电的顾客有 3 张优惠券，但只能用其中 1 张：

优惠券 1:如果标价至少是 50 元，那么标价降低 10%

优惠券 2:如果标价至少是 100 元，那么标价降低 20 美元

优惠券 3:标价超过 100 元的部分，降低 18%

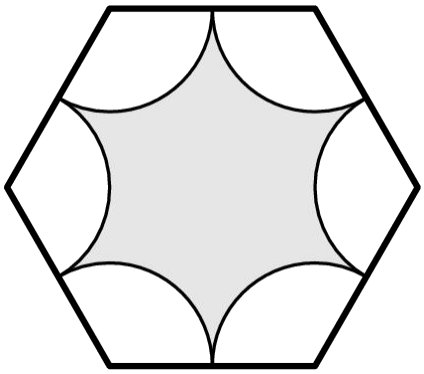
对于下面哪个标价，使用优惠券 1 会比使用优惠券 2 或 3 都划算？

$\textbf{(A)}\ \$179.95 \qquad \textbf{(B)}\ \$199.95 \qquad \textbf{(C)}\ \$219.95 \qquad \textbf{(D)}\ \$239.95 \qquad \textbf{(E)}\ \$259.95$

Problem 12

A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors as shown. The region inside the hexagon but outside the sectors is shaded as shown. What is the area of the shaded region?

一个正六边形的边长是 6。以每个顶点为圆心，3 为半径画弧，形成如图所示的扇形。在六边形内部但在扇形外部的区域如图中阴影部分所示，问阴影部分的面积是多少？

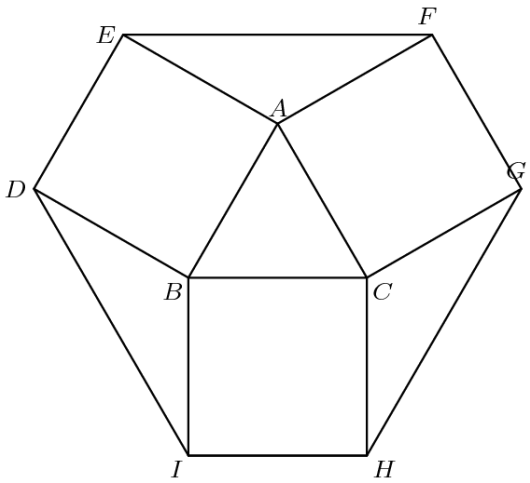


(A) $27\sqrt{3}-9\pi$ **(B)** $27\sqrt{3}-6\pi$ **(C)** $54\sqrt{3}-18\pi$ **(D)** $54\sqrt{3}-12\pi$ **(E)** $108\sqrt{3}-9\pi$

Problem 13

Equilateral $\triangle ABC$ has side length 1, and squares $ABDE, BCHI, CAFG$ lie outside the triangle. What is the area of hexagon $DEFGHI$?

等边三角形 *ABC* 的边长为 1，正方形 *ABDE*，*BCHI*，*CAFG* 在三角形外部，那么六边形 *DEFGHI* 的面积为多少？



(A) $\frac{12+3\sqrt{3}}{4}$ **(B)** $\frac{9}{2}$ **(C)** $3+\sqrt{3}$ **(D)** $\frac{6+3\sqrt{3}}{2}$ **(E)** 6

Problem 14

The y-intercepts, P and Q, of two perpendicular lines intersecting at the point $A(6,8)$ have a sum of zero. What is the area of $\triangle APQ$?

两条互相垂直交于点 $A(6,8)$ 的直线的 y 截距分别是 P 点和 Q 点，且它们的和为 0。问 $\triangle APQ$ 的面积是多少?

(A) 45 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

Problem 15

David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

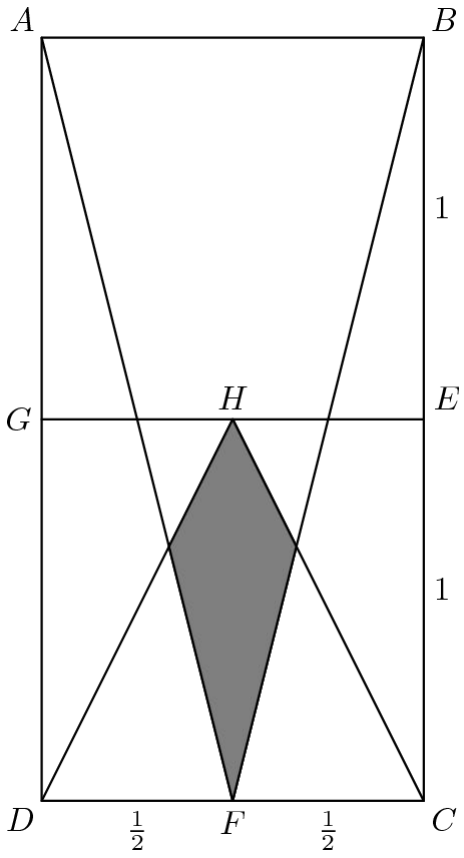
David 从他家开车到机场去坐飞机。他在第一小时开了 35 英里，但是意识到如果继续按照这个速度开车，会迟到 1 小时。剩下的路程他把车速增加了 15 英里每小时，结果提前了 30 分钟到达，从他家到机场有多少英里？

(A) 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

Problem 16

In rectangle $ABCD$, $AB = 1$, $BC = 2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?

在矩形 *ABCD* 中，*AB*=1 ，*BC*=2，点 *E*，*F*，*G* 分别是线段$\overline{BC}$，$\overline{CD}$和$\overline{AD}$的中点，点 *H* 是 *GE* 的中点，问阴影部分的面积是多少？



(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

Problem 17

Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?

掷了 3 个标准的六面骰子。问其中 2 个骰子上数字之和等于剩下那个骰子上数字的概率是多少?

(A) $\frac{1}{6}$ **(B)** $\frac{13}{72}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{5}{24}$ **(E)** $\frac{2}{9}$

Problem 18

A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?

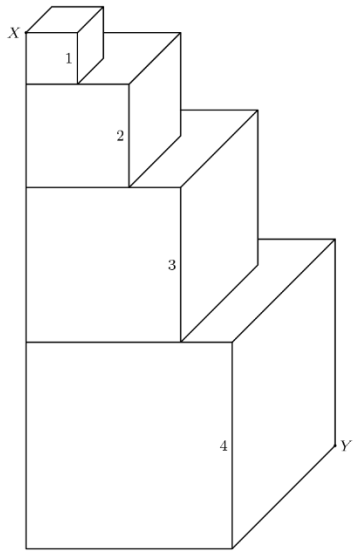
坐标平面内的一个正方形顶点的 y 坐标为 0，1，4，5。那么整个正方形的面积是多少?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 27$

Problem 19

Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of $\overline{XY}$ contained in the cube with edge length 3?

四个边长分别为 1，2，3，4 的正方体如图所示堆叠．线段$\overline{XY}$在边长为 3 的正方体内部的部分长度为多少?



$\textbf{(A)}\ \frac{3\sqrt{33}}{5} \qquad \textbf{(B)}\ 2\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{33}}{3} \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 3\sqrt{2}$

Problem 20

The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

$(8)(888\ldots8)$的乘积是一个各个位上数字之和为 1000 的整数，其中第二个乘数一共有 k 位。问 k 是多少?

$\textbf{(A)}\ 901 \qquad \textbf{(B)}\ 911 \qquad \textbf{(C)}\ 919 \qquad \textbf{(D)}\ 991 \qquad \textbf{(E)}\ 999$

Problem 21

Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

正整数 *a* 和 *b* 满足 $y = ax + 5$ 和$y = 3x + b$的图像交于 *x* 轴上同一点。问这个交点的所有可能的 *x* 坐标之和是多少？

(A) -20 **(B)** -18 **(C)** -15 **(D)** -12 **(E)** -8

Problem 22

In rectangle $ABCD$, $AB = 20$ and $BC = 10$. Let E be a point on $\overline{CD}$ such that $\angle CBE = 15°$. What is AE?

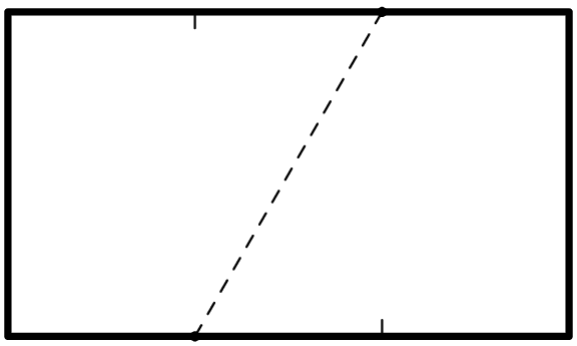
矩形 *ABCD* 中，*AB*=20， *BC*=10。*E* 是线段$\overline{CD}$上一点满足$\angle CBE = 15°$。则 *AE* 的长为多少？

(A) $\frac{20\sqrt{3}}{3}$ **(B)** $10\sqrt{3}$ **(C)** 18 **(D)** $11\sqrt{3}$ **(E)** 20

Problem 23

A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B : A$?

一张长方形的纸片，长是宽的 3 倍，面积为 *A*。纸片的两条对边被三等分，然后一条边的第一个等分点和对边的第二个等分点形成如图所示的虚线，之后把纸片沿着这条虚线对折铺平，形成一个新的形状，面积为 *B*。问 *B*：*A* 是多少?



(A) $1 : 2$ **(B)** $3 : 5$ **(C)** $2 : 3$ **(D)** $3 : 4$ **(E)** $4 : 5$

Problem 24

A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and, on the nth iteration, listing $n+3$ and skipping n. The sequence begins $1, 2, 3, 4, 6, 7, 8, 9, 10, 13$. What is the $500,000^{\text{th}}$ number in the sequence?

一个自然数数列是由这种方式构造的:先列出前 4 项，跳过 1 个，接着列出后面 5 个，然后跳过 2 个，列出 6 个，跳过 3 个，以此类推，在第 *n* 次时，列出 *n*＋3 个，跳过 *n* 个．这个数列的前几项为:1，2，3，4，6，7，8，9，10，13.这个数列的第 500，000 个数字是多少？

$\textbf{(A)}\ 996,506 \qquad \textbf{(B)}\ 996,507 \qquad \textbf{(C)}\ 996,508 \qquad \textbf{(D)}\ 996,509 \qquad \textbf{(E)}\ 996,510$

Problem 25

The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m,n) are there such that $1 \le m \le 2012$ and$5^n < 2^m < 2^{m+2} < 5^{n+1}$?

数字5^{867}位于2^{2013}和2^{2014}之间。有多少对整数（*m*，*n*），满足 1≤*m*≤2012 且 $5^n < 2^m < 2^{m+2} < 5^{n+1}$?

$\textbf{(A)}\ 278 \qquad \textbf{(B)}\ 279 \qquad \textbf{(C)}\ 280 \qquad \textbf{(D)}\ 281 \qquad \textbf{(E)}\ 282$

2014 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	C	E	B	C	A	B	D	C	B	C	C	C
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	C	E	D	B	A	D	E	E	C	A	B	