2012 AMC 10B

Problem 1

Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

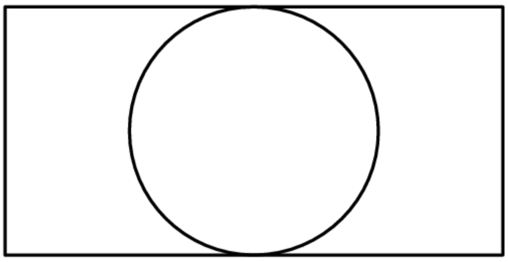
Pearl Creek 小学的每一个三年级救室都有 18 个学生和 2 只兔子，那么四个三年级教室的学生总数比兔子总数多多少个？

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 80$

Problem 2

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?

一个半径为 5 的圆和一个矩形内切，如果所示，矩形的长和宽之比为 2:1。问矩形的面积是多少?



$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 200$

Problem 3

The point in the xy-plane with coordinates $(1000, 2012)$ is reflected across the line $y = 2000$. What are the coordinates of the reflected point?

在坐标平面内，坐标为（1000，2012）的点关于直线 y=2000 作对称。那么对称点的坐标是多少?

$\textbf{(A)}\ (998,2012) \qquad \textbf{(B)}\ (1000,1988) \qquad \textbf{(C)}\ (1000,2024) \qquad \textbf{(D)}\ (1000,4012) \qquad \textbf{(E)}\ (1012,2012)$

Problem 4

When Ringo places his marbles into bags with 6 marbles per bag, he has 4 marbles left over. When Paul does the same with his marbles, he has 3 marbles left over. Ringo and Paul pool their marbles and place them into as many bags as possible, with 6 marbles per bag. How many marbles will be left over?

当 Ringo 把他的玻璃球放入包中，每个包放 6 个，最后还剩 4 个。当 Paul 也用同样方式把他自己的玻璃球放入包中后（每个包放 6 个），他最后还剩 3 个。Ringo 和 Paul 后来把他们各自的玻璃球都混在一起，然后放入尽可能多的包中，每个包仍然放 6 个玻璃球。问最后剩下多少个?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Problem 5

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of 27.50 dollars for dinner. What is the cost of her dinner without tax or tip in dollars?

Anna 在华盛顿一家餐馆正在吃晚餐，晚餐的税率是 10%。她给的小费是税前餐费的 15%，并且税费是根据给小费之前的餐费计算的。她这顿晚餐总共花了 27.5 美元。问如果不算税费和小费，晚餐原本费用是多少美元？

(A) 18 **(B)** 20 **(C)** 21 **(D)** 22 **(E)** 24

Problem 6

In order to estimate the value of $x-y$ where x and y are real numbers with $x>y>0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

x 和 y 都是实数且 $x>y>0$，为了估计$x-y$的值，Xiaoli 通过把 x 增加一定量使之五入到一个整数，通过把 y 减小同样的量使之四舍到一个整数，然后分别把 x 和 y 四舍五入后的值相减，问下面哪句话是对的？

（A）Her estimate is larger than-y丨她的估算比 x-y 大

（B）Her estimate is smaller than-y丨她的估算比 x-y 小

（C）Her estimate equalsx-y丨她的估算等于 x-y

（D）Her estimate equals y-x丨她的估算等于 y-x

（E）Her estimate is 0丨她的估算等于 0

Problem 7

For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

为了完成一项科学项目，Sammy 观察花栗鼠和松鼠在洞里藏橡果。花栗鼠在它所挖的每个洞里藏 3 颗橡果，而松鼠在它所挖的每个洞里藏 4 颗橡果。它们藏的橡果的总数一样多，但是松鼠所需要的洞比花栗鼠少 4 个。问花栗鼠藏了多少颗橡果?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

Problem 8

What is the sum of all integer solutions to $1 < (x-2)^2 < 25$?

不等式$1 < (x-2)^2 < 25$的所有整数解之和为多少？

(A) 10 **(B)** 12 **(C)** 15 **(D)** 19 **(E)** 25

Problem 9

Two integers have a sum of 26. When two more integers are added to the first two integers the sum is 41. Finally when two more integers are added to the sum of the previous four integers the sum is 57. What is the minimum number of odd integers among the 6 integers?

两个整数的和为 26，当在这两个整数的和的基础上，再加上另外两个整数，总和是 41。当在这前 4 个整数之和的基础上，再加上两个整数，和变成 57。问这 6 个整数中，奇数的个数最少是多少？

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Problem 10

How many ordered pairs of positive integers (M, N) satisfy the equation $\frac{M}{6}=\frac{6}{N}$?

有多少对正整数有序对（M，N），满足方程 $\frac{M}{6}=\frac{6}{N}$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 11

A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

一个甜点厨师给一周的每一天都准备了甜点，从周日开始，每天的甜点是蛋糕，馅饼，冰淇淋，或者布丁，不能连续两天都准备同一种甜点。因为周五有生日，所以周五必须是蛋糕，间一周一共有多少种不同的甜点菜单？

$\textbf{(A)}\ 729 \qquad \textbf{(B)}\ 972 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 2187 \qquad \textbf{(E)}\ 2304$

Problem 12

Point B is due east of point A. Point C is due north of point B. The distance between points A and C is $10\sqrt{2}$, and $\angle BAC = 45^\circ$. Point D is 20 meters due north of point C. The distance AD is between which two integers?

点 B 在点 A 的东边，点 C 在点 B 的北边，点 A 和点 C 的距离是$10\sqrt{2}$，且$\angle BAC = 45^\circ$。点 D 在点 C 的北边 20 米处，那么线段 AD 的长度在哪两个整数之间？

$\textbf{(A)}\ 30\ \text{and}\ 31 \qquad \textbf{(B)}\ 31\ \text{and}\ 32 \qquad \textbf{(C)}\ 32\ \text{and}\ 33 \qquad \textbf{(D)}\ 33\ \text{and}\ 34 \qquad \textbf{(E)}\ 34\ \text{and}\ 35$

Problem 13

It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

当电梯不运行时，Clea 从电梯走下来需要 60 秒，但是当电梯运行时，他从电梯走下来只要 24 秒，问当电梯运行时，如果 Clea 站在电梯上不动，从电梯下来需要多少秒？

(A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

Problem 14

Two equilateral triangles are contained in a square whose side length is $2\sqrt{3}$. The bases of these triangles are the opposite sides of the square, and their intersection is a rhombus. What is the area of the rhombus?

两个等边三角形位于一个边长为$2\sqrt{3}$的正方形的内部，这两个三角形的底是正方形的对边，它们相交的区域是一个菱形，那么这个菱形的面积是多少？

(A) $\frac{3}{2}$ (B) $\sqrt{3}$ (C) $2\sqrt{2}-1$ (D) $8\sqrt{3}-12$ (E) $\frac{4\sqrt{3}}{3}$

Problem 15

In a round-robin tournament with 6 teams, each team plays one game against each other team, and each game results in one team winning and one team losing. At the end of the tournament, the teams are ranked by the number of games won. What is the maximum number of teams that could be tied for the most wins at the end on the tournament?

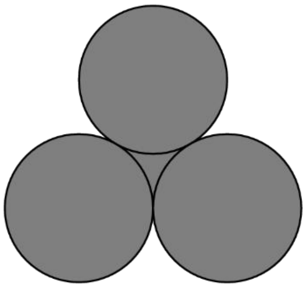
在一场有 6 支队伍的循环赛中，每支队伍都要和其他每支队伍打一场比赛，而且每场比赛的结果都是一胜一负。循环赛结束后，6 支队伍按照他们赢得比赛的场数排名。问排名并列第一的队伍最多可以是多少个？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 16

Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?

三个半径为 2 的圆相互外切。那么，这 3 个圆和这 3 个圆包围的中间区域所形成的如图所示的阴影部分的面积是多少？



(A) $10\pi + 4\sqrt{3}$ **(B)** $13\pi - \sqrt{3}$ **(C)** $12\pi + \sqrt{3}$ **(D)** $10\pi + 9$ **(E)** 13π

Problem 17

Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

Jesse 沿着半径为 12 的一个圆形纸盘的两条半径剪出了两个扇形，小的扇形的圆心角是 120 度。他用这两个扇形作为圆锥侧面做了两个圆锥。问小圆锥的体积和大圆锥的体积之比是多少？

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{5}}{5}$

Problem 18

Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive. For a person who does not have the disease, however, there is a 2% false positive rate--in other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease. Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p?

假设在某个特定的人群里，每 500 个人就有 1 个人有一种无症状的疾病，血液测试可以测出这种疾病。如果有人有这种病，那么血液测试结果总是阳性。如果一个人没有这种病，测试结果会有 2%的假阳性概率－换句话说，对于这类人，98%的时间测试结果会是阴性，但是会有 2%的时间，测试结果会是阳性，错误地认为这个人有这种疾病。p 表示随机从这个人群里选择的一个人，其测试结果是阳性，而实际上确实有这种疾病的概率，下面哪个最接近 p?

$\textbf{(A)}\ \frac{1}{98} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{1}{11} \qquad \textbf{(D)}\ \frac{49}{99} \qquad \textbf{(E)}\ \frac{98}{99}$

Problem 19

In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E, and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of $BFDG$?

在矩形 $ABCD$ 中， AB=6， AD=30，G 是线段$\overline{AD}$的中点。延长线段 AB 到 E，使得 BE=2，点 F 是$\overline{ED}$和$\overline{BC}$的交点。问 $BFDG$ 的面积是多少？

$\textbf{(A)}\ \frac{133}{2} \qquad \textbf{(B)}\ 67 \qquad \textbf{(C)}\ \frac{135}{2} \qquad \textbf{(D)}\ 68 \qquad \textbf{(E)}\ \frac{137}{2}$

Problem 20

Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

Bernardo 和 Silvia 玩有如下规则的游戏。从 0-999（包含 0 和 999）中选择一个整数，把它给 Bernardo。只要 Bernardo 拿到一个数，他就会把它乘以 2，然后把结果给 Silvia。只要 Silvia 得到一个数字，她就会把这个数字加 50，然后把结果再给 Bernardo。 赢得比赛的人是最后那个产生一个小于 1000 的数的人。令 *N* 表示可以使得 Bernardo 赢的最小的初始值，那么 *N* 的各个位上数字之和为多少？

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Problem 21

Four distinct points are arranged on a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

四个点在同一个平面上，满足连接它们的线段的长度是 *a*，*a*，*a*，*a*，*2a* 和 *b*。问 *b:a* 的比值是多少?

(A) $\sqrt{3}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** π

Problem 22

Let $(a_1, a_2, \dots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

$(a_1, a_2, \dots, a_{10})$是前 10 个正整数所组成的一列数字，满足对于每个 2≤*i*≤10，$a_i + 1$或者$a_i - 1$或者两者都排在a_i的前面。问一共有多少种这样的一列数?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** $181,440$ **(E)** $362,880$

Problem 23

A solid tetrahedron is sliced off a wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object?

一个边长为 1 的正方体形状的木头被一个平面切掉了一块四面体，这个平面通过正方体某一个面上不相邻的两个顶点，和与此面对面的面上跟这两个顶点都不相邻的某个顶点。被切掉的这个四面体被丢弃了，把正方体的剩余部分以切面作为底，放在桌子上。问这个物体的高度是多少？

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{2\sqrt{2}}{3}$ **(C)** 1 **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\sqrt{2}$

Problem 24

Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those girls but disliked by the third. In how many different ways is this possible?

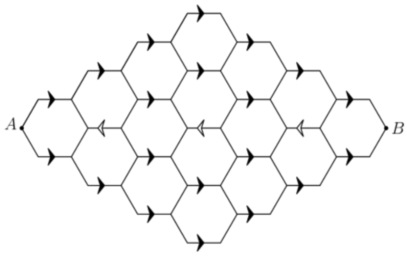
Amy，Beth 和 Jo 听四首不同的歌曲，并且讨论她们喜欢哪几首。不存在她们三个人都喜欢的歌，并且，对于这三对女孩中的每一对，至少有一首歌被这一对女孩都喜欢，但不会被第三个女孩喜欢，问她们一共有多少种可能的方式喜欢歌曲？

(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

Problem 25

A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?

一个虫子沿着如下图所示的六边形方格的边从 A 爬到 B。在标有箭头的线段爬行时，只能沿着箭头的方向爬行，并且虫子不能在同一根线段上爬行超过 1 次。一共有多少种不同的路径?



(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

2012 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	B	A	D	A	D	B	A	D	A	B	B
14	15	16	17	18	19	20	21	22	23	24	25	
D	D	A	C	C	C	A	A	B	D	B	E	