2011 AMC 10B

Problem 1

What is $\dfrac{2+4+6}{1+3+5} - \dfrac{1+3+5}{2+4+6}$

$\dfrac{2+4+6}{1+3+5} - \dfrac{1+3+5}{2+4+6}$的值是多少？

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \dfrac{5}{36} \qquad \textbf{(C)}\ \dfrac{7}{12} \qquad \textbf{(D)}\ \dfrac{147}{60} \qquad \textbf{(E)}\ \dfrac{43}{3}$

Problem 2

Josanna's test scores to date are $90, 80, 70, 60,$ and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

Josanna 目前的考试分数是 90，80，70，60 和 85。她的目标是靠下次考试将她的考试平均分提高至少 3 分，那么地下次至少需要考多少分才能完成这个目标？

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 85 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 95$

Problem 3

At a store, when a length is reported as x inches that means the length is at least $x - 0.5$ inches and at most $x + 0.5$ inches. Suppose the dimensions of a rectangular tile are reported as 2 inches by 3 inches. In square inches, what is the minimum area for the rectangle?

在某家商店，长度标为 x 英寸表示实际长度至少为$x - 0.5$英寸，至多为$x + 0.5$英寸。假设一块长方形瓷砖的尺寸标为 2 x 3 英寸。则在这个长方形瓷砖的最小面积是多少平方英寸？

$\textbf{(A)}\ 3.75 \qquad \textbf{(B)}\ 4.5 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8.75$

Problem 4

LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip, it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

LeRoy 和 Bemardo 一起出去旅游了一周，并且商定两人均摊费用。在这一周中，他们两人都支付了各种共同支出，例如汽油和租车费用。旅游结束后，发现 Latoy 花费 A 美元，Bernardo 花费 B 美元，这里 $A<B$。那么 LeRoy 需要支付 Bernardo 多少美元，这样能够保证他俩均摊了费用？

$\textbf{(A)}\ \frac{A+B}{2} \qquad \textbf{(B)}\ \frac{A-B}{2} \qquad \textbf{(C)}\ \frac{B-A}{2} \qquad \textbf{(D)}\ B-A \qquad \textbf{(E)}\ A+B$

Problem 5

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

Ron 在把 2 个正整数 a 和 b 相乘时，将两位数 a 的个位和十位数字弄反了，导致最后错误的乘积为 161。那么 a 和 b 相乘所得正确的乘积是多少?

$\textbf{(A)}\ 116 \qquad \textbf{(B)}\ 161 \qquad \textbf{(C)}\ 204 \qquad \textbf{(D)}\ 214 \qquad \textbf{(E)}\ 224$

Problem 6

On Halloween Casper ate $1/3$ of his candies and then gave 2 candies to his brother. The next day he ate $1/3$ of his remaining candies and then gave 4 candies to his sister. On the third day he ate his final 8 candies. How many candies did Casper have at the beginning?

在万圣节这天，Casper 吃掉了他的糖果总量的 1/3，并且给了他弟弟 2 个。第二天他吃掉了剩下糖果的$1/3$, 并且给了他妹妹 4 个。在第三天他把剩余的 8 颗糖果都吃完了。那么最初 Casper 有多少颗糖果？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 39 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 57 \qquad \textbf{(E)}\ 66$

Problem 7

The sum of two angles of a triangle is $6/5$ of a right angle, and one of these two angles is 30° larger than the other. What is the degree measure of the largest angle in the triangle?

一个三角形的个内角之和是直角度数的 6/5，并且这 2 个内角中其中一个角比另一个角大 30° 。那么这个三角形中最大的角的度数是多少度?

(A) 69 **(B)** 72 **(C)** 90 **(D)** 102 **(E)** 108

Problem 8

At a certain beach if it is at least $80^\circ F$ and sunny, then the beach will be crowded. On June 10 the beach was not crowded. What can be concluded about the weather conditions on June 10?

在某个沙滩，若气温至少是 80° F 且晴朗，则沙滩会挤满人。在 6 月 10 号这天，沙滩没有挤满人。则我们可以推断 6 月 10 号这天的天气是什么样的？

(A) The temperature was cooler than 80°F and it was not sunny.

温度低于 80° F 且不晴朗。

(B) The temperature was cooler than 80°F or it was not sunny.

温度低于 80° F 或者不晴朗。

(C) If the temperature was at least 80°F, then it was sunny.

若温度至少 80° F，则那天天气晴朗。

(D) If the temperature was cooler than 80°F, then it was sunny.

若温度低于 80° F，则那天天气晴朗。

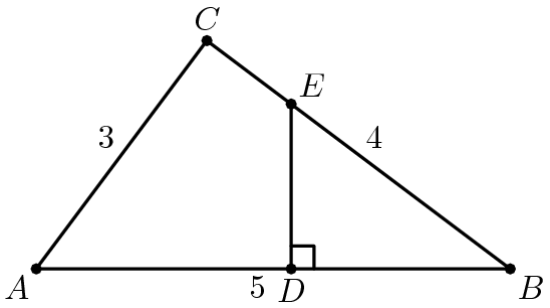
(E) If the temperature was cooler than 80°F, then it was not sunny.

若温度低于 80° F，则那天天气不晴朗。

Problem 9

The area of $\triangle EBD$ is one third of the area of $3-4-5$ $\triangle ABC$. Segment DE is perpendicular to segment AB. What is BD?

$\triangle EBD$的面积是$3-4-5$ $\triangle ABC$面积的三分之一。线段 *DE* 垂直线段 *AB*。那么 *BD* 长为多少？



(A) $\frac{4}{3}$ **(B)** $\sqrt{5}$ **(C)** $\frac{9}{4}$ **(D)** $\frac{4\sqrt{3}}{3}$ **(E)** $\frac{5}{2}$

Problem 10

Consider the set of numbers $\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

考虑集合$\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$。则集合里最大的元素和其他 10 个元素之和的比值最接近哪个整数？

(A) 1 **(B)** 9 **(C)** 10 **(D)** 11 **(E)**101

Problem 11

There are 52 people in a room. What is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

房间里有 52 人。使得命题“这个房间里至少有 *n* 个人的生日在同一个月”正确的 *n* 的最小值是多少？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 12

Problem 12

Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has a width of 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

Keiko 每天都以同样的速度绕跑道走一圈。跑道的两边是直线，两端是半圆。跑道的宽为 6 米，她沿着跑道的外沿走一圈比沿着内沿走多花 36 秒。那么 Keiko 的速度是多少米每秒？

$\textbf{(A)}\ \frac{\pi}{3} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Problem 13

Two real numbers are selected independently at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

从区间$[-20, 10]$内独立且随机的选择两个实数，那么这两个数的乘积大于 0 的概率是多少？

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

Problem 14

A rectangular parking lot has a diagonal of 25 meters and an area of 168 square meters. In meters, what is the perimeter of the parking lot?

一个矩形停车场的一条对角线长 25 米，面积为 168 平方米。那么停车场的周长是多少米？

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 58 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 68 \qquad \textbf{(E)}\ 70$

Problem 15

Let @ denote the "averaged with" operation: $a@b = (a+b)/2$. Which of the following distributive laws hold for all numbers x, y, and z?

令符号@表示“求平均”操作：$a@b = (a+b)/2$。下面哪个分配率适用于所有的数 x，y 和 z?

I. $x\ @\ (y+z) = (x\ @\ y) + (x\ @\ z)$

II. $x + (y\ @\ z) = (x+y)\ @\ (x+z)$

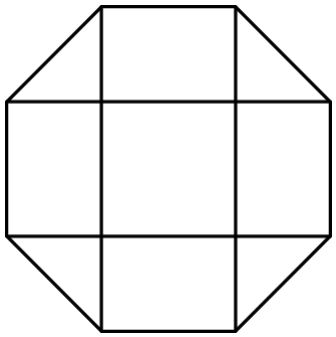
III. $x\ @\ (y\ @\ z) = (x\ @\ y)\ @\ (x\ @\ z)$

(A) I only **(B)** II only **(C)** III only **(D)** I and III only **(E)** II and III only

Problem 16

A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?

一种飞镖盘是一个被分割成如下图所示几个区域的正八边形。假设掷向飞镖盘的飞镖落在盘上任何点都是等可能的，那么飞镖落在位于中心位置的正方形的概率是多少？

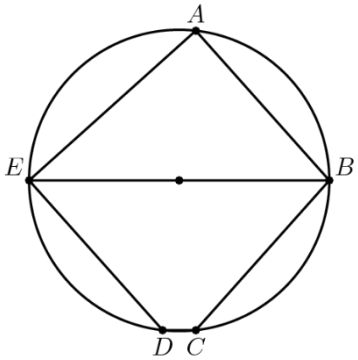


(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2-\sqrt{2}$

Problem 17

In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4:5$. What is the degree measure of angle BCD?

如图所示的圆中，直径$\overline{EB}$平行于$\overline{DC}$，$\overline{AB}$平行于$\overline{ED}$。角 *AEB* 和角 *ABE* 之比为 4:5。问角 *BCD* 是多少度？



$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 125 \qquad \textbf{(C)}\ 130 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 140$

Problem 18

Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

矩形 *ABCD* 中，边 *AB*=6， *BC*=3。点 *M* 在边 *AB* 上满足$\angle AMD = \angle CMD$。问$\angle AMD$的度数是多少？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Problem 19

What is the product of all the roots of the equation$\sqrt{5|x| + 8} = \sqrt{x^2 - 16}$.

$\sqrt{5|x| + 8} = \sqrt{x^2 - 16}$.所有根的乘积是多少?

$\textbf{(A)}\ -64 \qquad \textbf{(B)}\ -24 \qquad \textbf{(C)}\ -9 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 576$

Problem 20

Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

菱形 *ABCD* 的边长为 2， $\angle B = 120^\circ$ 。菱形内所有相对于顶点 *A*，*C*，*D* 来说，更靠近顶点 *B* 的点组成了区域 *R*。那么区域 *R* 的面积是多少？

$\textbf{(A)}\ \frac{\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{\sqrt{3}}{2} \qquad \textbf{(C)}\ \frac{2\sqrt{3}}{3} \qquad \textbf{(D)}\ 1+\frac{\sqrt{3}}{3} \qquad \textbf{(E)}\ 2$

Problem 21

Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

Brian 写下了 4 个整数$w > x > y > z$, 它们的和是 44。这些数两两之间的差的绝对值是 1，3，4，5，6 和 9。那么 *w* 的所有可能值的和是多少?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 93$

Problem 22

A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

一个底面是边长为 1 的正方形，侧面都是等边三角形的四棱锥内有一个正方体，正方体的一个面位于此棱锥的底面，而这个面的对面所有的棱都在棱锥的侧面上。则这个正方体的体积是多少？

$\textbf{(A)}\ 5\sqrt{2}-7 \qquad \textbf{(B)}\ 7-4\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{2}}{27} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{9} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{9}$

Problem 23

What is the hundreds digit of 2011^{2011}?

2011^{2011}的百位数字是多少?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 9$

Problem 24

A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $1/2 < m < a$. What is the maximum possible value of a?

xy 坐标系内的格点是指 *x* 和 *y* 均为整数的点（*x*，*y*）。对于 0<*x*≤100，满足$1/2 < m < a$的所有 *m*，都能使得$y = mx + 2$的图像不通过任何格点。那么 *a* 的最大可能值是多少？

$\textbf{(A)}\ \frac{51}{101}$ $\textbf{(B)}\ \frac{50}{99}$ $\textbf{(C)}\ \frac{51}{100}$ $\textbf{(D)}\ \frac{52}{101}$ $\textbf{(E)}\ \frac{13}{25}$

Problem 25

Let T_1 be a triangle with sides $2011, 2012,$ and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and $D, E,$ and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC and AC, respectively, then T_{n+1} is a triangle with side lengths $AD, BE,$ and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

T_1是个边长为2011，2012 和 2013 的三角形。对于 *n*≥1，若$T_n = \triangle ABC$且点 *D*，*E*，*F* 是$\triangle ABC$的内切圆分别和边AB, BC，和 *AC* 相切的切点，那么T_{n+1}是以线段$AD, BE,$和 *CF* 为三边的三角形（如果T_{n+1}存在的话）。问数列(T_n)中最后一个三角形的周长是多少？

$\textbf{(A)}\ \frac{1509}{8}$ $\textbf{(B)}\ \frac{1509}{32}$ $\textbf{(C)}\ \frac{1509}{64}$ $\textbf{(D)}\ \frac{1509}{128}$ $\textbf{(E)}\ \frac{1509}{256}$

2011 AMC 10B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	A	C	E	A	B	B	D	B	D	A	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	E	A	C	E	A	C	B	A	D	B	D	