2010 AMC 10A

Problem 1

Mary's top book shelf holds five books with the following widths, in centimeters: 6, $\frac{1}{2}$, 1, 2.5, and 10. What is the average book width, in centimeters?

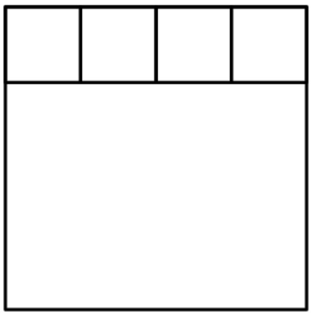
Mary 的顶层书架上放了 5 本书，它们的宽度（单位：厘米）分别是 6 和 10。问书本的平均宽度是多少厘米？

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Problem 2

Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?

4 个全等的正方形和 1 个矩形放置在一起形成一个大的正方形，如下图所示。问矩形的长是它的宽的几倍？



(A) $\frac{5}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

Problem 3

Tyrone had 97 marbles and Eric had 11 marbles. Tyrone then gave some of his marbles to Eric so that Tyrone ended with twice as many marbles as Eric. How many marbles did Tyrone give to Eric?

Tyrone 有 97 颗玻璃球，Eric 有 11 颗玻璃球。Tyrone 给了 Eric 若干颗玻璃球，最后 Tyrone 的玻璃球颗数是 Eric 的 2 倍。问 Tyrone 给了 Eric 多少颗玻璃球？

(A) 3 (B) 13 (C) 18 (D) 25 (E) 29

Problem 4

A book that is to be recorded onto compact discs takes 412 minutes to read aloud. Each disc can hold up to 56 minutes of reading. Assume that the smallest possible number of discs is used and that each disc contains the same length of reading. How many minutes of reading will each disc contain?

一本要刻录到光盘上的书需要 412 分钟才能完整读取出来。已知每张光盘最多可容纳 56 分钟的读取时间。假设使用的光盘数量尽可能少，并且每张光盘包含相同的读取时间。则每张光盘的读取时间为多少分钟？

(A) 50.2 (B) 51.5 (C) 52.4 (D) 53.8 (E) 55.2

Problem 5

The area of a circle is 24π，whose circumference is $k\pi$. What is the value of k?

一个圆的周长是24π，面积是$k\pi$，则 k 是多少？

(A) 6 (B) 12 (C) 24 (D) 36 (E) 144

Problem 6

For positive numbers x and y the operation $\spadesuit(x, y)$ is defined as

$$\spadesuit(x, y) = x - \frac{1}{y}$$

What is $\spadesuit(2, \spadesuit(2, 2))$?

x 和 y 都是正数，操作符$\spadesuit(x, y)$定义为$\spadesuit(x, y) = x - \frac{1}{y}$ 则$\spadesuit(2, \spadesuit(2, 2))$是多少？

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{5}{3}$ (E) 2

Problem 7

Crystal has a running course marked out for her daily run. She starts this run by heading due north for one mile. She then runs northeast for one mile, then southeast for one mile. The last portion of her run takes her on a straight line back to where she started. How far, in miles, is this last portion of her run?

Crystal 有一个每天跑步的跑步路线。她先朝正北方向跑了一英里，然后她向东北方向跑了一英里，再朝东南方向跑了一英里。最后则是沿着直线回到起点，这是跑步的最后一段。问她跑的最后一段距离是多少英里？

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 (E) $2\sqrt{2}$

Problem 8

Tony works 2 hours a day and is paid $\$0.50$ per hour for each full year of his age. During a six month period Tony worked 50 days and earned $\$630$. How old was Tony at the end of the six month period?

Tony 每天工作 2 小时，且他的时薪是按照这种方式计算的：他的年龄每增加一周岁，则每小时薪水增加 0.5 美元。在一段持续 6 个月的时间内，Tony 工作了 50 天，赚了 630 美元。问在这 6 个月末 Tony 是多少周岁？

(A) 9 (B) 11 (C) 12 (D) 13 (E) 14

Problem 9

A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x+32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

回环数是指具有这个特点的数:当将它的各位上数字从右向左重新排列后，形成的数和原来的数一样。例如 83438 就是个回环数。已知 x 和 x+32 分别是 3 位和 4 位回环数。那么 x 的各个位上数字之和是多少？

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

Problem 10

Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?

Marvin 在 2008 年（这是个闰年）5 月 27 号，周二过了生日。问后面哪一年他的生日是个周六？

(A) 2011 (B) 2012 (C) 2013 (D) 2015 (E) 2017

Problem 11

The length of the interval of solutions of the inequality $a \le 2x + 3 \le b$ is 10. What is $b - a$?

不等式$a \le 2x + 3 \le b$的解的区间长度为 10，则 b-a 是多少？

(A) 6 (B) 10 (C) 15 (D) 20 (E) 30

Problem 12

Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

Logan 为他的城镇按比例做了个模型。城镇的水塔高 40 米，顶部是一个能装 100，000 升的水的球体。Logan 的模型水塔能装 0.1 升的水，问 Logan 应该把他的模型塔做得多少米高？

(A) 0.04 **(B)** $\frac{0.4}{\pi}$ **(C)** 0.4 **(D)** $\frac{4}{\pi}$ **(E)** 4

Problem 13

Angelina drove at an average rate of 80 km/h and then stopped 20 minutes for gas. After the stop, she drove at an average rate of 100 km/h. Altogether she drove 250 km in a total trip time of 3 hours including the stop. Which equation could be used to solve for the time t in hours that she drove before her stop?

Angelina 以 80km/h 的平均速度驾驶，然后停了 20 分钟用于加油。加油完成后，她开车的平均速度为 100km/h。总的来说，整个旅程耗时 3 小时（这包括了加油时间），她共行驶了 250km。下面哪个方程可以用于解出她在停止前的驾驶时间 t（单位是小时）？

(A) $80t + 100\left(\frac{8}{3} - t\right) = 250$ (B) $80t = 250$ (C) $100t = 250$

(D) $90t = 250$ (E) $80\left(\frac{8}{3} - t\right) + 100t = 250$

Problem 14

Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

三角形 ABC 中， $AB = 2 \cdot AC$。点 D 和 E 分别在$\overline{AB}$和$\overline{BC}$上，满足$\angle BAE = \angle ACD$。令 F 为线段 AE 和 CD 的交点，假设$\triangle CFE$为等边三角形，那么$\angle ACB$是多少度？

$\textbf{(A)}\ 60^\circ \qquad \textbf{(B)}\ 75^\circ \qquad \textbf{(C)}\ 90^\circ \qquad \textbf{(D)}\ 105^\circ \qquad \textbf{(E)}\ 120^\circ$

Problem 15

In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

在一片神奇的沼泽中，有 2 种会说话的两栖动物：1 种是癞蛤蟆，它们总是说真话，还有 1 种是青蛙，它们总是说谎话。现在有 4 只两栖动物，它们分别是 Brian，Chris，LeRoy，和 Mike，都住在这片沼泽中，并有如下对话：

Brian 说:“Mike 和我是不同的种类。”

Chris 说：“LeRoy 是一只青蛙。”

LeRoy 说：“Chris 是一只青蛙。”

Mike 说：“我们这 4 只动物中，至少有 2 只是癞蛤蟆。”

问这 4 只两栖动物有几只是青蛙？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 16

Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

非退化$\triangle ABC$的边长均为整数，$\overline{BD}$是条角平分线， AD=3，且 DC=8。问它的周长最小可能值是多少？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 33 \qquad \textbf{(C)}\ 35 \qquad \textbf{(D)}\ 36 \qquad \textbf{(E)}\ 37$

Problem 17

A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

一个正方体的棱长为 3 英寸。每个面上都切出一个 2x2 英寸的正方形洞。每一个切口的边都平行于正方体的边，并且每个洞都贯穿整个正方体。问剩余立体图形的体积是多少立方英寸？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Problem 18

Bernardo randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1,2,3,4,5,6,7,8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

Bermardo 从集合｛1，2，3，4，5，6，7，8，9｝中随机选择 3 个不同的数，并把它们降序排列形成一个 3 位数。Silvia 从集合｛1，2，3，4，5，6，7，8｝中随机选择 3 个不同的数，也把它们降序排列形成一个 3 位数。问 Bermardo 的数比 Silvia 的数大的概率是多少？

$\textbf{(A)}\ \frac{47}{72} \qquad \textbf{(B)}\ \frac{37}{56} \qquad \textbf{(C)}\ \frac{2}{3} \qquad \textbf{(D)}\ \frac{49}{72} \qquad \textbf{(E)}\ \frac{39}{56}$

Problem 19

Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

等角六边形 ABCDEF 的边长为 AB=CD=EF=1，且 BC=DE=FA=r。已知$\triangle ACE$的面积是六边形面积的 70％。那么 r 的所有可能值之和是多少？

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

Problem 20

A fly trapped inside a cubical box with side length 1 meter decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?

一只苍蝇被困在一个棱长为 1 米的正方体盒子里。为了打发时间，它决定将盒子的每个顶点都访问一遍。它的起点和终点是同一个顶点，其他顶点都只访问一次。它每次都沿着直线从一个顶点飞到或者爬到其他顶点。问它的路径总长度最多是多少米?

(A) $4+4\sqrt{2}$ (B) $2+4\sqrt{2}+2\sqrt{3}$ (C) $2+3\sqrt{2}+3\sqrt{3}$

(D) $4\sqrt{2}+4\sqrt{3}$ (E) $3\sqrt{2}+5\sqrt{3}$

Problem 21

The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a?

多项式$x^3 - ax^2 + bx - 2010$有 3 个正整数根。问 a 的最小可能值是多少？

(A) 78 (B) 88 (C) 98 (D) 108 (E) 118

Problem 22

Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

从某个圆上选 8 个点，并且这 8 个点两两相连形成弦。不存在 3 条弦交于圆内同一点。问圆内形成多少个这样的三角形，满足三角形的 3 个顶点均在圆内？

(A) 28 (B) 56 (C) 70 (D) 84 (E) 140

Problem 23

Each of 2010 boxes in a line contains a single red marble, and for $1 \leq k \leq 2010$, the box in the $k\text{th}$ position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

一条直线上有 2010 个盒子，每个盒子内都装有一颗红色玻璃球，并且对于 $1 \leqslant k \leqslant 2010$，在第 k 个位置的盒子也包含 k 颗白色玻璃球。Isabella 从第一个盒子开始，连续地依次从每个盒子里随机抽取一颗玻璃球。当她第一次抽到红色玻璃球后，她便停止。令$P(n)$表示当 Isabella 抽了恰好 n 颗玻璃球后停止的概率。$P(n) < \frac{1}{2010}$的最小的 n 是多少?

(A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

Problem 24

The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

90!的最后两个非零数字组成的数是 n，问 n 是多少？

(A) 12 **(B)** 32 **(C)** 48 **(D)** 52 **(E)** 68

Problem 25

Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{array}{ccccc} & & & & 55 \\ 55 & - & 7^2 & = & 6 \\ 6 & - & 2^2 & = & 2 \\ 2 & - & 1^2 & = & 1 \\ 1 & - & 1^2 & = & 0 \end{array}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?

Jim 从某个正整数 *n*，得到一组数字。下一个数字是由当前的数字减去小于或等于当前数字的最大平方数而得，直到最后得到 0 为止。例如，如果 Jim 一开始的数字是 *n*=55,那么他得到的这组数字包含 5 个数：

$$\begin{array}{ccccc} & & & & 55 \\ 55 & - & 7^2 & = & 6 \\ 6 & - & 2^2 & = & 2 \\ 2 & - & 1^2 & = & 1 \\ 1 & - & 1^2 & = & 0 \end{array}$$

使得 Jim 获得的一组数有 8 个的最小数为 N。问 N 的个位是多少？

(A) 1　　(B) 3　　(C) 5　　(D) 7　　(E) 9

2010 AMC 10A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	B	D	B	E	C	C	D	E	E	D	C	A
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	D	B	A	B	E	D	A	A	A	A	B	